PART VII

Answers to Selected Exercises, and Review Problems

Answers to Selected Exercises

Exercise 3.1 (random financing). (i) The investors' breakeven condition is

$$xI - A \leqslant x p_{\mathrm{H}}(R - R_{\mathrm{b}}).$$

Because the NPV is negative if the entrepreneur has an incentive to shirk, R_{b} must satisfy

$$(\Delta p)R_{\mathrm{b}} \geqslant B.$$

The investors' breakeven condition (which will be satisfied with equality under a competitive capital market) is then

$$x\left[p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) - I\right] \geqslant -A$$

or

$$x\bar{A} \leqslant A.$$

(ii) The NPV is equal to

$$U_{\mathrm{b}} = x(p_{\mathrm{H}}R - I)$$

and so maximizing U_{b} is tantamount to maximizing x. Hence,

$$x^* = \frac{A}{\bar{A}}.$$

The probability that the project is undertaken grows from 0 to 1 as the borrower's net worth grows from 0 to $\bar{A}$.

Exercise 3.2 (impact of entrepreneurial risk aversion). (i) When $p_{\mathrm{H}} < 1$, the entrepreneur must receive at least c_0 in the case of failure, because the probability of failure is positive even in the case of good behavior. Because of risk neutrality above c_0, it is optimal to give the entrepreneur exactly c_0 in the case of failure. Let R_{b} denote the reward in the case of success.

The incentive constraint is

$$(\Delta p)(R_{\mathrm{b}} - c_0) \geqslant B. \qquad \text{(IC)}$$

The pledgeable income is

$$p_{\mathrm{H}}R - (1 - p_{\mathrm{H}})c_0 - p_{\mathrm{H}} \min_{\{\mathrm{IC}\}} R_{\mathrm{b}} = p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) - c_0.$$

To allow financing, this pledgeable income must exceed $I - A$. Hence, $\bar{A} = I + c_0 - p_{\mathrm{H}}(R - B/\Delta p)$.

When $p_{\mathrm{H}} = 1$, the pledgeable income is then $p_{\mathrm{H}}R$ (if $c_0 > 0$, deviations can be punished harshly by giving the entrepreneur, say, 0 in the case of failure).

(ii) Let $R_{\mathrm{b}}^{\mathrm{S}}$ and $R_{\mathrm{b}}^{\mathrm{F}}$ denote the rewards in the cases of success and failure, respectively. The incentive constraint is

$$(\Delta p)[u(R_{\mathrm{b}}^{\mathrm{S}}) - u(R_{\mathrm{b}}^{\mathrm{F}})] \geqslant B.$$

The optimal contract solves

$$\max U_{\mathrm{b}} = p_{\mathrm{H}}u(R_{\mathrm{b}}^{\mathrm{S}}) + (1 - p_{\mathrm{H}})u(R_{\mathrm{b}}^{\mathrm{F}})$$

s.t.

$$p_{\mathrm{H}}R - p_{\mathrm{H}}R_{\mathrm{b}}^{\mathrm{S}} - (1 - p_{\mathrm{H}})R_{\mathrm{b}}^{\mathrm{F}} \geqslant I - A,$$

$$(\Delta p)[u(R_{\mathrm{b}}^{\mathrm{S}}) - u(R_{\mathrm{b}}^{\mathrm{F}})] \geqslant B,$$

and (if limited liability is imposed)

$$R_{\mathrm{b}}^{\mathrm{F}} \geqslant 0.$$

It must also be the case that the solution to this program exceeds the utility, $u(A)$, obtained by the entrepreneur if the project is not financed. The entrepreneur's incentive compatibility constraint is binding; otherwise, the solution to this program would give full insurance to the entrepreneur, which would violate the incentive compatibility condition. We refer to Holmström[1] and Shavell[2] for general considerations on this moral-hazard problem.

Exercise 3.3 (random private benefits). (i) $B^* = p_{\mathrm{H}}(R - r_1)$.

1. Holmström, B. 1979. Moral hazard and observability. *Bell Journal of Economics* 10:74–91.
2. Shavell, S. 1979. Risk sharing and incentives in the principal and agent relationship. *Bell Journal of Economics* 10:55–73.

(ii) The investors' expected income is

$$p_{\mathrm{H}}^2 \frac{r_1(R-r_1)}{R} I = \frac{B^*(p_{\mathrm{H}}R - B^*)}{R} I.$$

The borrowing capacity is such that this expected income is equal to the investors' initial investment, $I - A$. Thus

$$I = kA,$$

where

$$k = \frac{1}{1 - B^*(p_{\mathrm{H}}R - B^*)/R}.$$

The borrowing capacity is maximized for

$$B^* = \tfrac{1}{2} p_{\mathrm{H}} R,$$

or, equivalently,

$$r_1 = \tfrac{1}{2} R.$$

(iii) Using the fact that investors break even, the entrepreneur's expected utility is

$$\left(p_{\mathrm{H}} \frac{B^*}{R} R + \int_{B^*}^{R} \frac{B}{R}\, \mathrm{d}B\right) I = \frac{p_{\mathrm{H}}B^* + \frac{1}{2}R - (B^*)^2/2R}{1 - B^*(p_{\mathrm{H}}R - B^*)/R} A.$$

At the optimum,

$$\tfrac{1}{2} p_{\mathrm{H}} R < B^* < p_{\mathrm{H}} R.$$

Recall that $B^* = p_{\mathrm{H}}R$ maximizes the return per unit of investment as it eliminates shirking, while $B^* = \frac{1}{2}p_{\mathrm{H}}R$ maximizes borrowing capacity.

(iv) When B is verifiable, the entrepreneur's expected utility is still

$$\left(p_{\mathrm{H}}B^* + \frac{R}{2} - \frac{(B^*)^2}{2R}\right) I.$$

For a given B^*, the contract should specify

$$r_1(B) \begin{cases} = R - B/p_{\mathrm{H}} & \text{if } B < B^* \text{ (recall that } p_{\mathrm{L}} = 0), \\ > R - B/p_{\mathrm{H}} & \text{if } B > B^*. \end{cases}$$

The maximal investment is then

$$I = \frac{A}{1 - p_{\mathrm{H}}B^* + (B^*)^2/2R}.$$

Borrowing capacity is maximized at $B^* = p_{\mathrm{H}}R$. Because this threshold also maximizes per-unit expected income, it is clearly optimal overall.

Exercise 3.4 (product-market competition and financing). (i) Because the two projects are statistically independent, there is no point making an entrepreneur's reward contingent on the outcome of the other firm's performance. (Technically, this result is a special case of the "sufficient statistics" results of Holmström[3] and Shavell[4]. This result states that an agent's reward should be contingent only on variables that the agent can control—a sufficient statistic for the vector of observable variables relative to effort—and not on extraneous noise.) So, let $R_{\mathrm{b}}^{\mathrm{S}}$ and $R_{\mathrm{b}}^{\mathrm{F}}$ denote an entrepreneur's reward in the cases of success and failure. As usual,

$$(\Delta p)(R_{\mathrm{b}}^{\mathrm{S}} - R_{\mathrm{b}}^{\mathrm{F}}) \geqslant B \quad \text{and} \quad R_{\mathrm{b}}^{\mathrm{F}} = 0.$$

Let $x \in [0,1]$ denote the probability that the rival firm invests. Then the expected income is

$$p_{\mathrm{H}}[x p_{\mathrm{H}} D + (1 - x p_{\mathrm{H}}) M].$$

The pledgeable income is equal to this expression minus $p_{\mathrm{H}}B/\Delta p$.

At best, the other firm is not financed, and $R = M$ in the case of success. The threshold $\underline{A}$ is given by

$$I - \underline{A} = p_{\mathrm{H}}\left(M - \frac{B}{\Delta p}\right).$$

(ii) At worst, the rival firm is financed. So, the expected return in the case of success is

$$p_{\mathrm{H}} D + (1 - p_{\mathrm{H}}) M.$$

So,

$$I - \bar{A} = p_{\mathrm{H}}\left(p_{\mathrm{H}} D + (1 - p_{\mathrm{H}}) M - \frac{B}{\Delta p}\right).$$

(iii) One of the firms gets funding while the other does not (obvious). There also exists a third, mixed-strategy equilibrium, in which each firm gets funded with positive probability.

(iv) If only one firm receives financing, then

$$R_{\mathrm{b}}^{\mathrm{F}} = c_0$$

(as long as $p_{\mathrm{H}} < 1$, so that there is always a probability of failing even when the entrepreneur works), and

$$R_{\mathrm{b}}^{\mathrm{S}} = c_0 + \frac{B}{\Delta p},$$

which yields the minimum net worth given in the statement of the question.

(v) Suppose now that both entrepreneurs receive financing. Consider the following reward scheme for

3. Holmström, B. 1979. Moral hazard and observability. *Bell Journal of Economics* 10:74–91.

4. Shavell, S. 1979. Risk sharing and incentives in the principal and agent relationship. *Bell Journal of Economics* 10:55–73.

the entrepreneur:

$$R_b < c_0 \quad \text{if the firm fails and the rival firm succeeds,}$$
$$R_b = c_0 \quad \text{otherwise.}$$

There is no longer moral hazard: as long as the other entrepreneur works, shirking yields probability Δp that the other entrepreneur succeeds while this entrepreneur fails (recall that the two technologies are perfectly correlated), resulting in a large (infinite) punishment. If

$$D > M - \frac{B}{\Delta p},$$

then product-market competition facilitates financing! Correlation enables benchmarking provided that both firms secure financing.

Exercise 3.5 (continuous investment and decreasing returns to scale). (i) The incentive constraint is, as in the model of Section 3.4,

$$(\Delta p)R_b \geqslant BI. \tag{IC}$$

The pledgeable income is

$$p_H\Big[R(I) - \min_{\{IC\}} R_b\Big] = p_H\Big[R(I) - \frac{BI}{\Delta p}\Big].$$

Thus the entrepreneur selects I to solve

$$\max \text{NPV} = \max U_b = p_H R(I) - I$$
$$\text{s.t.}$$
$$p_H\Big[R(I) - \frac{BI}{\Delta p}\Big] \geqslant I - A. \tag{BB}$$

Clearly, if $I = I^*$ satisfies (BB) (A is high), then it solves this program. The shadow price of the budget constraint is then $\mu = 0$.

So suppose A is small enough that (BB) is not satisfied at $I = I^*$. Then I is determined by (BB) (since the objective function is concave). In that region, by the envelope theorem

$$\frac{dU_b}{dA} = v = [p_H R'(I) - 1]\frac{dI}{dA} = \frac{1}{(p_H B/\Delta p)/(p_H R' - 1) - 1}.$$

So v decreases with A.

Exercise 3.6 (renegotiation and debt forgiveness). (i) Suppose that $R_b < BI/(\Delta p)$.

In the absence of renegotiation, the entrepreneur will shirk and obtain utility

$$BI + p_L R_b,$$

and the lender's expected revenue is

$$p_L(RI - R_b).$$

Renegotiation must be mutually advantageous. So a necessary condition for renegotiation is that total surplus increases. A renegotiation toward a stake $\hat{R}_b < BI/(\Delta p)$ does not affect surplus and thus is a mere redistribution of wealth between the investors and the entrepreneur. So renegotiation, if it happens, must yield stake

$$\hat{R}_b \geqslant \frac{BI}{\Delta p}$$

for the entrepreneur. It constitutes a Pareto-improvement if the following two conditions are satisfied:

$$p_H \hat{R}_b \geqslant BI + p_L R_b$$

and

$$p_H(RI - \hat{R}_b) \geqslant p_L(RI - R_b).$$

The second inequality, together with the incentive constraint, implies that

$$(\Delta p)RI - p_H\frac{BI}{\Delta p} + p_L R_b \geqslant 0.$$

Conversely, if this condition is satisfied, then the two parties can find an $\hat{R}_b$ that makes them both better off.

Note that the standard assumptions

$$p_H\Big[RI - \frac{BI}{\Delta p}\Big] \geqslant I - A$$

and

$$I \geqslant p_L RI + BI$$

imply that

$$(\Delta p)RI - p_H\frac{BI}{\Delta p} + A - BI \geqslant 0.$$

So, if $A > BI$ and R_b is small enough, the condition for renegotiation may not be satisfied.

(ii) The "project" consists in creating incentives for the entrepreneur. It creates NPV equal to $(\Delta p)RI$, does not involve any new investment, and the entrepreneur can bring an amount of money $\hat{A} \equiv p_L R_b$ that is the forgone expected income.

For this fictitious project, the pledgeable income is

$$(\Delta p)RI - p_{\mathrm{H}}\frac{BI}{\Delta p}$$

and the investors' outlay is

$$0 - \hat{A}.$$

Hence, it is "financed" if and only if

$$(\Delta p)RI - p_{\mathrm{H}}\frac{BI}{\Delta p} \geqslant -p_{\mathrm{L}}R_{\mathrm{b}}.$$

Exercise 3.7 (strategic leverage). (i) • The NPV, if the project is funded, is

$$(p_{\mathrm{H}} + \tau)R - I(\tau).$$

So, if $A \geqslant A^*$, $\tau = \tau^*$.

• For $A < A^*$, the pledgeable income can be increased by reducing τ below τ^*:

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left[(p_{\mathrm{H}}+\tau)\left(R-\frac{B}{\Delta p}\right)-[I(\tau)-A]\right] = R-\frac{B}{\Delta p}-I'(\tau).$$

Let τ^{**} be defined by

$$I'(\tau^{**}) = R - \frac{B}{\Delta p}.$$

The pledgeable income decreases with τ for $\tau \geqslant \tau^{**}$. The borrower can raise funds if and only if $A > A^{**}$, with

$$(p_{\mathrm{H}} + \tau^{**})\left(R - \frac{B}{\Delta p}\right) = I(\tau^{**}) - A^{**}.$$

The quality of investment increases with A (for $A > A^{**}$) and is flat beyond A^*. For $A \in [A^{**}, A^*]$,

$$[p_{\mathrm{H}} + \tau(A)]\left[R - \frac{B}{\Delta p}\right] = I(\tau(A)) - A.$$

For $A \geqslant A^*$, $\tau(A) = \tau^*$.

(ii) • Define $\hat{\tau}$ by

$$I'(\hat{\tau}) = [1 - (p_{\mathrm{H}} + \hat{\tau})]R.$$

($\hat{\tau}$ maximizes a firm's NPV given that the other firm's choice is $\hat{\tau}$.) Borrower i's incentive compatibility constraint is $(\Delta p)(1-q_j)R_{\mathrm{b}} \geqslant B$, where R_{b} is her reward in the case of income R. So the pledgeable income is

$$(p_{\mathrm{H}} + \tau)\left[(1 - q_j)R - \frac{B}{\Delta p}\right].$$

$(\hat{\tau}, \hat{\tau})$ is a symmetric Nash equilibrium if and only if

$$(p_{\mathrm{H}} + \hat{\tau})\left[[1 - (p_{\mathrm{H}} + \hat{\tau})]R - \frac{B}{\Delta p}\right] \geqslant I(\hat{\tau}) - A.$$

This equation yields $\hat{A}$.

• "Natural monopoly case." Let $\tau(A)$ be defined as in subquestion (i). Consider a candidate equilibrium in which borrower 1 selects $\tau(A)$ and borrower 2 does not raise funds. That is,

$$A \leqslant \min_{\tau}\left\{I(\tau)-(p_{\mathrm{H}}+\tau)\left[(1-(p_{\mathrm{H}}+\tau(A)))R-\frac{B}{\Delta p}\right]\right\}.$$

(iii) • $(\tilde{q}, \tilde{q})$ is a symmetric Nash equilibrium for $A = \tilde{A}$.

• By choosing $q_1 = \tilde{q} + \varepsilon$, borrower 1 deters entry by borrower 2.

Exercise 3.8 (equity multiplier and active monitoring). (i) See Section 3.4.

(ii) Suppose that monitoring at level c is to be induced. Two incentive compatibility constraints must be satisfied:

$$(\Delta p)R_{\mathrm{m}} \geqslant cI \quad \text{and} \quad (\Delta p)R_{\mathrm{b}} \geqslant b(c)I.$$

Because there is no scarcity of monitoring capital, the monitor contributes I_{m} to the project and breaks even:

$$I_{\mathrm{m}} = p_{\mathrm{H}}R_{\mathrm{m}} - cI = p_{\mathrm{H}}\frac{c}{\Delta p}I - cI.$$

The equity multiplier, k, is given by

$$p_{\mathrm{H}}(R - R_{\mathrm{b}} - R_{\mathrm{m}})I = I - A - I_{\mathrm{m}}$$

or

$$p_{\mathrm{H}}\left[R - \frac{b(c) + c}{\Delta p}\right]I = I - A - p_{\mathrm{H}}\frac{c}{\Delta p}I + cI,$$

that is,

$$I = k(c)A,$$

where

$$k(c) = \frac{1}{1 + c - p_{\mathrm{H}}[R - b(c)/\Delta p]} = \frac{1}{1 - \rho_0 + c + (p_{\mathrm{H}}/\Delta p)[b(c) - B]}.$$

The project's NPV (which includes the monitoring cost) is equal to

$$\rho_1 I - I - cI = (\rho_1 - 1 - c)k(c)A.$$

The borrower maximizes $(\rho_1 - 1 - c)k(c)$ since the other parties receive zero utility and she therefore receives the project's NPV.

Exercise 3.9 (concave private benefit). (i) Suppose that the NPV per unit of investment is positive:

$$p_{\mathrm{H}}R > 1$$

(otherwise there is no investment).

The entrepreneur's utility is equal to the NPV,

$$U_b = (p_H R - 1)I,$$

and so the entrepreneur chooses the highest investment that is consistent with the investors' breakeven constraint

$$p_H\left(RI - \frac{B(I)}{\Delta p}\right) = I - A.$$

Because $\lim_{I\to\infty} B'(I) = B$ and $p_H(R - (B/\Delta p)) < 1$, this upper limit indeed exists.

(ii) The shadow price is given by

$$v = \frac{dU_b}{dA} = (p_H R - 1)\frac{dI}{dA} = \frac{1}{(p_H B'(I)/(p_H R - 1)) - 1}.$$

Hence v increases with A (since $B'' < 0$ and $dI/dA > 0$).

Exercise 3.10 (congruence, pledgeable income, and power of incentive scheme). (i) Either $R_b \geqslant B/(\Delta p)$ and the entrepreneur always behaves well. The NPV is

$$\text{NPV}^1 = p_H R - I + (1 - x)B$$

and the financing condition

$$p_H\left(R - \frac{B}{\Delta p}\right) \geqslant I - A. \tag{1}$$

Or $R_b < B/(\Delta p)$. The NPV is then

$$\text{NPV}^2 = x(p_L R + B) + (1 - x)(p_H R + B) - I < \text{NPV}^1,$$

and the financing condition ($R_b = 0$ then maximizes the pledgeable income) is

$$[x p_L + (1 - x)p_H]R \geqslant I - A. \tag{2}$$

The pledgeable income is increased only if x is sufficiently low. The high-powered incentive scheme is always preferable if (1) is satisfied; otherwise, the parties may content themselves with a low-powered scheme (provided (2) is satisfied).

(ii) Suppose that the menu offers (R_b^S, R_b^F) when interests are divergent and $(\hat{R}_b^S, \hat{R}_b^F)$ when interests are aligned. The state (divergent/congruent) is not observed by the investors and so this menu must be incentive compatible (the entrepreneur must indeed prefer the incentive scheme tailored to the state of nature she faces).

The interesting case is when the incentive scheme in the divergent state is incentive compatible ($(\Delta p)\times(R_b^S - R_b^F) \geqslant B$; otherwise, setting all rewards equal to 0 is obviously optimal).

In the congruent state, the entrepreneur must not pretend interests are divergent, and so

$$p_H \hat{R}_b^S + (1 - p_H)\hat{R}_b^F \geqslant p_H R_b^S + (1 - p_H)R_b^F.$$

So one might as well take $\hat{R}_b^S = R_b^S$ and $\hat{R}_b^F = R_b^F$. This choice yields incentive compatibility in the congruent state and maximizes the pledgeable income.

Exercise 3.11 (retained-earnings benefit). (i) Let us assume away any discounting for notational simplicity. The assumption on $\underline{B}^2$ implies that retained earnings are always needed to finance the second project, as

$$p_H^2\left(R^2 - \frac{B^2}{\Delta p^2}\right) < I^2 \quad \text{for all } B^2.$$

The borrower's utility is, as a function of date-1 earnings R_b^1,

$$U_b(R_b^1) = \begin{cases} R_b^1 & \text{if the second project is not financed,} \\ R_b^1 + \text{NPV}^2 & \text{otherwise,} \end{cases}$$

where

$$\text{NPV}^2 = p_H^2 R^2 - I^2$$

is independent of B^2.

Let $\hat{R}_b^1(B^2)$ denote the required level of retained earnings when the date-2 private benefit turns out to be B^2:

$$p_H^2\left(R^2 - \frac{B^2}{\Delta p^2}\right) = I^2 - \hat{R}_b^1(B^2).$$

This equation also defines a threshold $\hat{B}^2(R_b^1)$.

Thus, the expected utility is

$$E[U_b(R_b^1)] = R_b^1 + F(\hat{B}^2(R_b^1))[\text{NPV}^2].$$

The shadow value of retained earnings is therefore

$$\mu = \frac{d[E[U_b(R_b^1)]]}{dR_b^1} = 1 + f(\hat{B}^2(R_b^1))\left[\frac{d\hat{B}^2}{dR_b^1}\right][\text{NPV}^2].$$

(ii) The date-1 incentive compatibility constraint is

$$(\Delta p^1)[R_b^1 + F(\hat{B}^2(R_b^1))[\text{NPV}^2]] \geqslant B^1.$$

The pledgeable income,

$$p_H^1\Big[R^1 - \min_{\{\text{IC}^1\}} R_b^1\Big],$$

is therefore larger than in the absence of a second project. It is therefore more likely to exceed $I^1 - A^1$, where A^1 is the entrepreneur's initial wealth.

Exercise 3.12 (investor risk aversion and risk premia). (i) This condition says that the risk-free rate is normalized at 0. In other words, investors are willing to lend 1 unit at date 0 against a safe return of 1 unit at date 1.

(ii) With a competitive capital market, the financing condition becomes

$$p_H q_S R_l \geqslant I - A.$$

With a risk-neutral entrepreneur, the incentive compatibility constraint is unchanged:

$$(\Delta p) R_b \geqslant B.$$

Thus, enough pledgeable income can be harnessed provided that

$$p_H\left[R - \frac{B}{\Delta p}\right] \geqslant \frac{I - A}{q_S}. \tag{1}$$

Comparing condition (1) with condition (3.3) in Chapter 3, we conclude that obtaining financing is easier for a countercyclical firm than for a procyclical one, ceteris paribus.

(iii) The entrepreneur maximizes her utility subject to the investors' being willing to lend

$$\max_{\{R_b^F, R_b^S\}} \{p_H R_b^S + (1 - p_H) R_b^F\} \tag{2}$$

s.t.

$$q_S p_H (R - R_b^S) + q_F (1 - p_H)(-R_b^F) \geqslant I - A, \tag{3}$$

$$(\Delta p)(R_b^S - R_b^F) \geqslant B, \tag{4}$$

$$R_b^F \geqslant 0. \tag{5}$$

Letting μ_1, μ_2, and μ_3 denote the shadow prices of the constraints, the first-order conditions are

$$p_H[1 - \mu_1 q_S] + \mu_2(\Delta p) = 0 \tag{6}$$

and

$$(1 - p_H)[1 - \mu_1 q_F] - \mu_2(\Delta p) + \mu_3 = 0. \tag{7}$$

• First, note that for $q_S \neq q_F$ at least one of constraints (4) and (5) must be binding: if $\mu_2 = \mu_3 = 0$, (6) and (7) cannot be simultaneously satisfied.

• Conversely, (4) and (5) cannot be simultaneously binding, except when condition (1) is satisfied with exact equality.

• Suppose that constraint (4) is not binding ($\mu_2 = 0$), which, from what has gone before, implies that $R_b^F = 0$. Then $\mu_1 = 1/q_S$, and (7) can be satisfied only if

$$q_F > q_S.$$

• In contrast, suppose that constraint (5) is not binding ($\mu_3 = 0$). Constraints (6) and (7) taken together imply that

$$q_S > q_F.$$

To sum up, the maximum punishment result ($R_b^F = 0$) carries over to procyclical firms, because the incentive effect compounds with the "marginal rates of substitution" effect (the investors value income in the case of failure relatively more compared with the entrepreneur). But it does not in general hold for countercyclical firms. Then the investors care more about the payoff in the case of success, and the entrepreneur should keep marginal incentives equal to $B/\Delta p$ and select $R_b^F > 0$ (since the firm's income is equal to 0 in the case of failure, this requires the firm to hoard some claim at date 0 so as to be able to pay the entrepreneur even in the case of failure).

Entrepreneurial risk aversion changes the incentive constraint (4) and the objective function (2). It may be the case that $R_b^F > 0$ even for a procyclical firm.

Exercise 3.13 (lender market power). (i) If $A \geqslant I$, then the "borrower" does not need the lender and just obtains the NPV ($U_b = V$). So let us assume that $A < I$. The lender must respect two constraints. First, the standard incentive compatibility constraint:

$$(\Delta p) R_b \geqslant B. \tag{IC$_b$}$$

Second, her net utility must be nonnegative:

$$U_b = p_H R_b - A \geqslant 0. \tag{IR$_b$}$$

The lender maximizes

$$U_l = p_H[R - R_b] - (I - A)$$

subject to these two constraints.

Let us first ignore (IC$_b$). The lender sets $R_b = A/p_H$ and thus

$$U_b = 0.$$

The lender appropriates the entire surplus ($U_l = V$) as long as $R_b = A/p_H$ satisfies the incentive

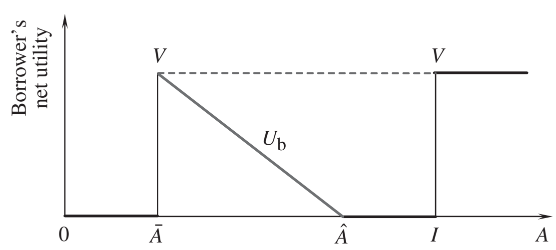

Figure 1

constraint, or

$$(\Delta p)\frac{A}{p_H} \geqslant B \quad \Longleftrightarrow \quad A \geqslant \hat{A}.$$

For $A \in [\bar{A}, \hat{A})$, the lender cannot capture the borrower's surplus without violating the incentive constraint; then the borrower's net utility

$$U_b = p_H \frac{B}{\Delta p} - A$$

is decreasing in A.

Lastly, the lender is willing to lend as long as

$$U_l = V - U_b \geqslant 0 \quad \text{or} \quad A \geqslant \bar{A}.$$

The borrower's net utility is as represented in Figure 1.

The borrower is "better off" (from the relationship) if she is either very rich (she does not need the lender) or poor (she cannot be expropriated by the lender)—although, of course, not too poor!

(ii) The lender solves

$$\max U_l = p_H(RI - R_b) - (I - A)$$

s.t.

$$(\Delta p)R_b \geqslant BI, \tag{IC$_b$}$$

$$p_H R_b \geqslant A. \tag{IR$_b$}$$

If (IC_b) were not binding, (IR_b) would have to be binding (U_l is decreasing in R_b) and

$$U_l = (p_H R - 1)I$$

would yield $I = \infty$, violating (IC_b), a contradiction.

If (IR_b) were not binding, (IC_b) would have to be binding, and

$$U_l = \left(p_H\left(R - \frac{B}{\Delta p}\right) - 1\right)I + A,$$

and so $I = 0 = R_b$, contradicting (IR_b).

Hence, the two constraints are binding, and so

$$I = \frac{1}{p_H B/\Delta p} A.$$

Recall that, in the presence of a competitive market,

$$I^* = \frac{1}{1 - p_H(R - B/\Delta p)} A,$$

and so

$$I < I^*.$$

With variable-size investment, lender market power leads to a contraction of investment.

Exercise 3.14 (liquidation incentives). (i) Technically, the realization of y is a "sufficient statistic" for inferring the effort chosen by the entrepreneur. Rewarding the entrepreneur as a function not only of y, but also of the realization of the final profit amounts to introducing into the incentive scheme noise over which the entrepreneur has no control. (We leave it to the reader to start with a general incentive scheme and then show that without loss of generality the reward can be made contingent on y only.)

Second, it is optimal to liquidate if and only if $y = \underline{y}$. Hence, one can define expected profits:

$$R^S \equiv \bar{y}R \quad \text{and} \quad R^F \equiv L,$$

where "success" ("S") now refers to a good signal, "failure" ("F") to a bad signal, and R^S and R^F denote the associated continuation profits.

We are now in a position to apply the analysis of Section 3.2. Let R_b^S denote the entrepreneur's reward in the case of a good signal ($y = \bar{y}$) and 0 that in the case of a bad signal. Incentive compatibility requires that

$$(\Delta p)R_b^S \geqslant B.$$

The NPV is

$$U_b \equiv p_H \bar{y}R + (1 - p_H)L - I,$$

and the pledgeable income is

$$\mathcal{P} \equiv p_H \bar{y}R + (1 - p_H)L - p_H \frac{B}{\Delta p}.$$

Financing is then feasible provided that $A \geqslant \bar{A}$, where

$$p_H\left(\bar{y}R - \frac{B}{\Delta p}\right) + (1 - p_H)L = I - \bar{A}.$$

(ii) Truth telling by the entrepreneur requires that

$$\bar{y}R_b \geqslant L_b \geqslant \underline{y}R_b.$$

The entrepreneur's other incentive compatibility constraint (that relative to effort) is then

$$(\Delta p)(\bar{y}R_b - L_b) \geqslant B.$$

The investors' payoff is then

$$p_H\bar{y}(R - R_b) + (1 - p_H)(L - L_b).$$

As expected, it is highest when L_b and R_b are as small as is consistent with the incentive constraints:

$$L_b = \underline{y}R_b \quad \text{and} \quad (\Delta p)(\bar{y}R_b - L_b) = B.$$

And so the pledgeable income is

$$p_H\bar{y}(R - R_b) + (1 - p_H)(L - L_b)$$

for these values of L_b and R_b. Simple computations show that the financing condition amounts to

$$p_H\bar{y}R + (1 - p_H)L - [p_H\bar{y} + (1 - p_H)\underline{y}]\frac{B}{(\Delta p)(\Delta y)} \geqslant I - A$$

or

$$A \geqslant \bar{A} + \underline{y}\frac{B}{(\Delta p)(\Delta y)}.$$

Exercise 3.15 (project riskiness and credit rationing). The managerial minimum reward (consistent with incentive compatibility) is the same for both variants:

$$\frac{B}{\Delta p^A} = \frac{B}{\Delta p^B}.$$

And so the investors' breakeven condition can be written (with obvious notation) as

$$I - A \leqslant p_H^A\left(R^A - \frac{B}{\Delta p}\right) \quad \text{for variant A}$$

and

$$I - A \leqslant p_H^B\left(R^B - \frac{B}{\Delta p}\right) \quad \text{for variant B}.$$

Because $p_H^A > p_H^B$, the safer project (project A) is financed for a smaller range of cash on hand A. That is, the safe project is more prone to credit rationing. Intuitively, the nonpledgeable income is higher for a safe project, since the entrepreneur has a higher chance to be successful and thus to receive the incentive payment $B/\Delta p$.

This, however, assumes that good behavior is needed for funding either variant. Let us relax this assumption. Good behavior boosts the pledgeable income (as well as the NPV, for that matter) more when the payoff in the case of success is high, that is, for the risky project. Thus, suppose that the following conditions hold:

$$I - A > p_H^B\left(R^B - \frac{B}{\Delta p}\right),$$
$$I - A > p_L^B R^B,$$
$$I - A \leqslant p_L^A R^A,$$
$$I < p_L^A R^A + B.$$

The first two inequalities state that the risky variant cannot receive financing whether good behavior or misbehavior is induced by the managerial compensation scheme (note, for example, that the second inequality is automatically satisfied if p_H^B is close to its lowest feasible value Δp). The third states that the risky project generates enough pledgeable income when the cash-flow rights are allocated entirely to investors. Finally, the fourth inequality guarantees that the safe project's NPV is positive.

To check that these inequalities are not inconsistent, assume, for example, that $A = 0$ and $p_L^A R^A = I$ (or just above); then

$$p_L^B R^B = \frac{p_L^B/p_L^A}{p_H^B/p_H^A} I < I.$$

Lastly, for B large enough, the first inequality is satisfied. We conclude that the risky project may be more prone to credit rationing if high-powered incentives are not necessarily called for.

Exercise 4.15 investigates a different notion of project risk, in which a safe project yields a higher liquidation value and a lower long-term payoff and is less prone to credit rationing than a risky project.

Exercise 3.16 (scale versus riskiness tradeoff). The risky project's NPV is

$$U_b^r = (x\rho_1 - 1)I.$$

The investors' breakeven condition can be written as

$$x\rho_0 I = I - A.$$

And so

$$U_b^r = \frac{x\rho_1 - 1}{1 - x\rho_0}A = \frac{\rho_1 - 1/x}{1/x - \rho_0}A.$$

Note that this is the same formula as obtained in Section 3.4.2, except that the expected cost of bringing

1 unit of investment to completion is $1/x$ rather than 1.

Turn now to the safe project. The NPV is then

$$U_b^s = (\rho_1 - X)I,$$

and the investors' breakeven condition is

$$\rho_0 I = XI - A.$$

Hence,

$$U_b^s = \frac{\rho_1 - X}{X - \rho_0} A.$$

The expected cost of bringing 1 unit of investment to completion is now X.

Thus the safe project is strictly preferred to the risky one if and only if

$$X < \frac{1}{x} \quad \text{or} \quad xX < 1.$$

Exercise 3.17 (competitive product market interactions). The representative firm's investment must satisfy

$$p_H\left[PR - \frac{B}{\Delta p}\right] i \geqslant i - A, \tag{1}$$

since the manager's reward in the case of success, R_b, must satisfy

$$(\Delta p) R_b \geqslant Bi.$$

The representative entrepreneur wants to borrow up to her borrowing capacity as long as the NPV per unit of investment is positive:

$$p_H PR \geqslant 1. \tag{2}$$

In equilibrium $i = I$ and $P = P(p_H RI)$. Let I^* (the optimal level from an individual firm's viewpoint) be given by

$$p_H RP^* = 1 \quad \text{and} \quad P^* = P(p_H RI^*).$$

Two cases must therefore be considered, depending on whether A is (a) large or (b) small:

(a) if

$$p_H[P^*R - B/\Delta p]I^* \geqslant I^* - A,$$

then the borrowing constraint is not binding and $I = I^*$;

(b) if

$$p_H[P^*R - B/\Delta p]I^* < I^* - A,$$

then (1) is binding, and so

$$I = \frac{A}{1 - p_H[RP(p_H RI) - B/\Delta p]}.$$

Exercise 3.18 (maximal incentives principle in the fixed-investment model). Recall that, because the investors break even, the entrepreneur's expected payoff when the project is financed is nothing but the project's NPV. The entrepreneur's expected payoff is therefore independent of the way the investment is financed. The financing structure just serves the purpose of guaranteeing good behavior by the entrepreneur. Let R_b^S and R_b^F denote the (nonnegative) rewards of the borrower in the cases of success (R^S) and failure (R^F), respectively. The incentive constraint can be written as

$$(\Delta p)(R_b^S - R_b^F) \geqslant B. \tag{IC$_b$}$$

This constraint implies that setting R_b^F at its minimum level (0) provides the entrepreneur with maximal incentives. So, the incentive constraint becomes

$$(\Delta p) R_b^S \geqslant B.$$

The pledgeable income is equal to total expected income minus the borrower's minimum stake consistent with incentives to behave:

$$p_H R^S + (1 - p_H)R^F - p_H \frac{B}{\Delta p} = p_H\left(R - \frac{B}{\Delta p}\right) + R^F.$$

Thus the project is financed if and only if

$$p_H\left(R - \frac{B}{\Delta p}\right) \geqslant I - (A + R^F). \tag{1}$$

As one would expect, the minimum income R^F plays the same role as cash or collateral. It is really part of the borrower's net worth.

The optimum contract can be implemented through a *debt contract*: let D, $R^F < D < R^S$, be defined by

$$p_H D + (1 - p_H)R^F = I - A. \tag{IR$_l$}$$

That is, the borrower owes D to the lenders. In the case of failure (R^F), the borrower defaults and the lenders receive the firm's cash, R^F. Equation (IR$_l$) then guarantees that the lenders break even.

In this fixed-investment version of the model, the debt contract is, however, in general not uniquely optimal: a small reward $R_b^F > 0$ for the borrower in the case of failure would still be consistent with (IC$_b$) and (IR$_l$) as long as condition (IR$_l$) is satisfied with strict inequality. By contrast, the standard debt contract is uniquely optimal in the variable-investment version of the model as it maximizes the borrower's borrowing capacity (see Section 3.4.3).

Exercise 3.19 (balanced-budget investment subsidy and profit tax). The total investment subsidy is sI and the profit tax tRI. Budget balance then requires
$$p_{\mathrm{H}} tRI = sI.$$
The amount of income that is pledgeable to investors is
$$p_{\mathrm{H}}\left[R - tR - -\frac{B}{\Delta p}\right]I,$$
and so the breakeven constraint is
$$p_{\mathrm{H}}\left[(1-t)R - \frac{B}{\Delta p}\right]I = (1-s)I - A.$$
Adding up the two equalities yields
$$p_{\mathrm{H}}\left[R - \frac{B}{\Delta p}\right]I = I - A$$
or
$$I = \frac{A}{1-\rho_0}.$$
Finally, the entrepreneur receives the NPV, $(\rho_1 - 1)I$, since both the investors and the government make no surplus.

Exercise 3.20 (variable effort, the marginal value of net worth, and the pooling of equity). (i) Let R_{b} denote the entrepreneur's reward in the case of success. The entrepreneur is residual claimant when she does not need to borrow:
$$R_{\mathrm{b}} = R.$$
And so she maximizes
$$\max_{p}\{pR - \tfrac{1}{2}p^2 - I\}$$
yielding
$$p = R$$
and
$$U_{\mathrm{b}} = \tfrac{1}{2}R^2 - I > 0.$$
(ii) More generally,
$$p = R_{\mathrm{b}}.$$
The investors' breakeven condition is
$$p(R - R_{\mathrm{b}}) \geqslant I - A$$
or
$$R_{\mathrm{b}}(R - R_{\mathrm{b}}) \geqslant I - A.$$
Only the region $R_{\mathrm{b}} \geqslant \frac{1}{2}R$ is relevant: were R_{b} to be smaller than $\frac{1}{2}R$, then $\hat{R}_{\mathrm{b}} = R - R_{\mathrm{b}}$ would yield the same pledgeable income, but a higher utility to the entrepreneur.

The highest pledgeable income is obtained when $R_{\mathrm{b}} = \frac{1}{2}R$. Thus a necessary condition for financing is that $A \geqslant A_1$, where
$$\tfrac{1}{4}R^2 = I - A_1.$$
It must further be the case that the project's NPV be positive. That is, for the (maximum) value of R_{b} satisfying
$$R_{\mathrm{b}}(R - R_{\mathrm{b}}) = I - A,$$
then
$$U_{\mathrm{b}} = R_{\mathrm{b}}R - I - \tfrac{1}{2}R_{\mathrm{b}}^2 - A \geqslant 0.$$
So, using the breakeven constraint to rewrite the NPV, let
$$U_{\mathrm{b}} = V(A) = \max_{\{R_{\mathrm{b}}\}}\{R_{\mathrm{b}}R - \tfrac{1}{2}R_{\mathrm{b}}^2 - I\}$$
s.t.
$$R_{\mathrm{b}}(R - R_{\mathrm{b}}) \geqslant I - A.$$
This yields the shadow price of equity, $V'(A)$:
$$V'(A) = [R - R_{\mathrm{b}}(A)]\left[\frac{\mathrm{d}R_{\mathrm{b}}(A)}{\mathrm{d}A}\right] > 0,$$
where $R_{\mathrm{b}}(A)$ is given by the investors' breakeven condition. For $A > I$, we can define $V(A) = (\frac{1}{2}R^2) - I$. And so $V'(A) = 0$ (note that we discuss net utilities, so the no-agency-cost benchmark is a shadow price of cash on hand equal to 0; this benchmark is equal to 1 for gross utilities). When $A > I$, the entrepreneur is residual claimant and exerts the socially optimal effort. For $A < I$, $V'(A) > 0$, but $V'(I) = 0$: a local increase in the entrepreneur's compensation just below R has only a second-order effect.

Furthermore,
$$V''(A) < 0.$$
Let $A_2 < I$ satisfy
$$V(A_2) = 0.$$
Then
$$\bar{A} = \max\{A_1, A_2\}.$$
(iii) Let $I \equiv I_{\mathrm{L}}$. That is, we fix I_{L} and the corresponding $V(\cdot)$ function. In the absence of an *ex ante* arrangement between the two entrepreneurs, each receives a *net* utility:
$$\tfrac{1}{2}V(A)$$
(the gross utility is $\frac{1}{2}(V(A) + A)$). For, because $R_{\mathrm{b}}R - \frac{1}{2}R_{\mathrm{b}}^2$ is concave, it is optimal for both to have

the same reward if they both invest. Thus the strategy consisting in (a) pooling cash on hand, (b) investing, and (c) setting identical reward schemes and investment, yields, for each entrepreneur,

$$V(A - \tfrac{1}{2}(I_H - I_L)).$$

Alternatively, the two can pool resources but only the low-investment-cost project will be funded. The expected net utility of each is then

$$\tfrac{1}{2}V(\max(2A, I_L)),$$

since, if $2A \geqslant I_L$, the low-investment-cost entrepreneur is residual claimant.

Note that

$$\tfrac{1}{2}V(\max(2A, I_L)) > \tfrac{1}{2}V(A),$$

so pooling is always optimal.

The lucky entrepreneur cross-subsidizes the unlucky entrepreneur if and only if

$$V(A - \tfrac{1}{2}(I_H - I_L)) > \tfrac{1}{2}V(\max(2A, I_L)).$$

The unlucky entrepreneur cross-subsidizes the lucky one if this inequality is violated. Finally, because

$$V(A) > \tfrac{1}{2}V(\max(2A, I_L)),$$

the cross-subsidization is from the lucky to the unlucky for I_H below some threshold.

Exercise 3.21 (hedging or gambling on net worth?). (i) Letting R_b denote the entrepreneur's stake in success (and 0 in failure), the incentive compatibility constraint is

$$(\Delta p)R_b \geqslant B.$$

Financing is feasible if and only if

$$p_H\left(R - \frac{B}{\Delta p}\right) \geqslant I - A.$$

The entrepreneur's date-1 *gross* utility is

$$[p_H R - I] + [A - \bar{A}] \quad \text{if } A \geqslant \bar{A}$$

and

$$A \quad \text{if } A < \bar{A}.$$

- If $A_0 \geqslant \bar{A}$, the entrepreneur's date-0 expected gross utility is

$$U_b^h = [p_H R - I] + A_0$$

if she hedges.

By contrast, and letting $F(\varepsilon)$ denote the cumulative distribution of ε, her expected utility becomes

$$\begin{aligned} U_b^g &= [1 - F(\bar{A} - A_0)][[p_H R - I] + m^+(\bar{A})] \\ &\quad + F(\bar{A} - A_0)m^-(\bar{A}) \\ &< U_b^h, \end{aligned}$$

where

$$m^+(\bar{A}) \equiv E[A \mid A \geqslant \bar{A}],$$
$$m^-(\bar{A}) \equiv E[A \mid A < \bar{A}],$$
$$[1 - F(\bar{A} - A_0)]m^+(\bar{A}) + F(\bar{A} - A_0)m^-(\bar{A}) = A_0.$$

- If $A_0 < \bar{A}$, then

$$U_b^h = A_0 < U_b^g.$$

(ii) *Ex post* the entrepreneur chooses p so as to solve

$$\max_{\{p\}}\{pR_b - \tfrac{1}{2}p^2\},$$

and so

$$p = R_b.$$

The pledgeable income is

$$\mathcal{P} = R_b(R - R_b)$$

and the NPV, i.e., the entrepreneur's expected *net* utility, in the case of financing is

$$U_b = R_b R - I.$$

Without loss of generality, assume that $R_b \geqslant \frac{1}{2}R$ (if $R_b < \frac{1}{2}R$, $\hat{R}_b = R - R_b$ yields the same $\mathcal{P}$ and a higher U_b).

Assume that $I - A_0 < \frac{1}{4}R^2$. This condition means that the entrepreneur can receive funding if she hedges (the highest pledgeable income is reached for $R_b = \frac{1}{2}R$). She also receives funding even in the absence of hedging provided that the support of ε is small enough (the lower bound is smaller than $\frac{1}{4}R^2 - (I - A_0)$ in absolute value). Let

$$V(A) \equiv R_b(A)R - I,$$

where $R_b(A)$ is the largest root of

$$R_b(R - R_b) = I - A.$$

One has

$$\frac{dV}{dA} = R\frac{dR_b}{dA} = \frac{R}{2R_b(A) - R} > 0$$

and

$$\frac{d^2V}{dA^2} = -\frac{2R}{(2R_b(A) - R)^2}\frac{dR_b}{dA} < 0.$$

Hence, V is concave and so

$$V(A_0) > E[V(A_0+\varepsilon)].$$

The entrepreneur is better off hedging.

(iii) The investment is given by the investors' breakeven condition:

$$p_{\rm H}\left[RI - \frac{B(I)}{\Delta p}\right] = I - A.$$

This yields investment $I(A)$, with $I' > 0$ and $I'' < 0$ if $B'' > 0$, $I'' > 0$ if $B'' < 0$. The *ex ante* utility is

$$U_{\rm b}^{\rm h} = (p_{\rm H}R - 1)E[I(A_0+\varepsilon)]$$

in the absence of hedging. And so $U_{\rm b}^{\rm h} > U_{\rm b}^{\rm g}$ if $B'' > 0$ and $U_{\rm b}^{\rm h} < U_{\rm b}^{\rm g}$ if $B'' < 0$.

(iv) When the profit is unobservable by investors, there is no pledgeable income and so

$$I = A.$$

And so

$$U_{\rm b}^{\rm h} = R(A_0) \quad \text{and} \quad U_{\rm b}^{\rm g} = E[R(A_0+\varepsilon)] < R(A_0)$$

since R is concave.

(v) Quite generally, in the absence of hedging the realization of ε generates a distribution $G(I)$ over investment levels $I = I(\varepsilon)$ and over cash used in the project $\mathcal{A}(\varepsilon) \leqslant A_0 + \varepsilon$ such that

$$\mathcal{P}(I(\varepsilon)) \geqslant I(\varepsilon) - \mathcal{A}(\varepsilon),$$

where $\mathcal{P}$ is the pledgeable income. And so

$$E[\mathcal{P}(I)] \geqslant E[I] - A_0.$$

Drawing I from distribution $G(\cdot)$ regardless of the realization of ε and keeping $A_0 - E[\mathcal{A}(\varepsilon)]$ makes the entrepreneur as well off.

In general, the entrepreneur can do strictly better by insulating her investment from the realization of ε (in the constant-returns-to-scale model of Section 3.4, though, she is indifferent between hedging and gambling).

Consider, for example, the case $A_0 < \bar{A}$ in subquestion (i). Then we know that gambling is optimal. The probability that the project is financed is

$$1 - F(\bar{A} - A_0) \quad \text{and} \quad [1 - F(\bar{A} - A_0)]\bar{A} < A_0.$$

This last inequality states that there is almost surely "unused cash": either $A_0 + \varepsilon < \bar{A}$ and then there is no investment, or $A_0 + \varepsilon > \bar{A}$ and then there is "excess cash" $[A_0 + \varepsilon - \bar{A}]$.

Consider therefore the date-0 contract in which the date-1 income $r = A_0 + \varepsilon$ is pledged to investors. The probability of funding is then X, which allows investors to break even:

$$A_0 = X\left[I - p_{\rm H}\left(R - \frac{B}{\Delta p}\right)\right] = X\bar{A}.$$

Clearly,

$$X > 1 - F(\bar{A} - A_0),$$

and so the entrepreneur's date-0 expected gross utility has increased from

$$[1 - F(\bar{A} - A_0)](p_{\rm H}R - I) + A_0$$

to

$$X(p_{\rm H}R - I) + A_0.$$

Of course, this is not quite a fair comparison, since we have allowed random funding under hedging and not under gambling. But, because there is excess cash in states of nature in which $A > \bar{A}$, the same result would hold even if we allowed for random funding under gambling: when $A < \bar{A}$, the project could be funded with probability $x(A) = A/\bar{A}$. The total probability of funding under gambling would then be

$$\int_0^{\bar{A}-A_0} \frac{A\,{\rm d}F(A - A_0)}{\bar{A}} + [1 - F(\bar{A} - A_0)]$$
$$< \frac{\int_0^{\bar{A}-A_0} A\,{\rm d}F(A - A_0) + \int_{\bar{A}-A_0}^{\infty} A\,{\rm d}F(A - A_0)}{\bar{A}} = \frac{A_0}{\bar{A}}.$$

For more on liquidity and risk management, see Chapter 5.

Exercise 4.1 (maintenance of collateral and asset depletion just before distress). (i) When $c = 0$ (no moral hazard on maintenance), the pledgeable income is equal to (A plus)

$$p_{\rm H}\left(R - \frac{B}{\Delta p}\right).$$

Consider $c > 0$. First, suppose that the entrepreneur receives $R_{\rm b}$ in the case of success, and $r_{\rm b}$ in the case of good maintenance. That is, the two incentives are not linked together. The IC constraints are

$$(\Delta p)R_{\rm b} \geqslant B \quad \text{and} \quad r_{\rm b} \geqslant c.$$

The pledgeable income is (A plus)

$$p_{\rm H}\left(R - \frac{B}{\Delta p}\right) - c.$$

However, and as in Diamond's (1984) model (see Section 4.2), it is optimal to link the two incentives. Let us look for conditions that guarantee that the entrepreneur both exerts effort to raise the probability of success and maintains the collateral. We just saw that it is optimal to reward the entrepreneur only if the project is successful *and* the asset has been maintained. Let $R_b > 0$ denote this reward. There are three potential incentive compatibility constraints:

- {work, maintain} $\succeq$ {shirk, maintain}

$$p_H R_b - c \geqslant p_L R_b - c + B$$

or

$$(\Delta p) R_b \geqslant B.$$

- {work, maintain} $\succeq$ {shirk, do not maintain}

$$p_H R_b - c \geqslant B.$$

Note that this second constraint does not bind if the first constraint is satisfied, since by assumption $p_L B/(\Delta p) \geqslant c$.

- {work, maintain} $\succeq$ {work, do not maintain}

$$p_H R_b - c \geqslant 0.$$

This third constraint is not binding either.

The necessary and sufficient condition for financing is

$$p_H\left(R - \frac{B}{\Delta p}\right) \geqslant I - A,$$

and the NPV is

$$U_b = [p_H R - I] + [A - c].$$

(ii) The decision over whether to maintain the collateral now depends on the realization of the signal about the eventual outcome of the project. The entrepreneur stops maintaining the asset when learning that the project will fail. When no signal accrues, the conditional probability of success (assuming that the entrepreneur has chosen probability of success $p \in \{p_L, p_H\}$) is

$$\frac{p}{p + (1-p)(1-\xi)}.$$

The borrower maintains the asset if and only if

$$\frac{p}{p + (1-p)(1-\xi)}(R_b + A) \geqslant c.$$

The *ex ante* incentive compatibility condition (relative to the choice of p) is then (for c not too large)

$$\begin{aligned} &p_H(R_b + A - c) + (1 - p_H)(1-\xi)(-c) \\ &\qquad \geqslant p_L(R_b + A - c) + (1 - p_L)(1-\xi)(-c) + B. \end{aligned}$$

The interpretation of the term $(\Delta p)\xi c$ in the inequality in the statement of question (ii) is that if the entrepreneur works, she reduces the probability of receiving a signal that enables her to avoid maintenance benefitting the lenders.

(iii) • Suppose, first, that the entrepreneur does not pledge the assets. Then the condition for financing is the familiar one (with the value of collateral, A, being nonpledgeable to investors):

$$p_H\left(R - \frac{B}{\Delta p}\right) \geqslant I.$$

• If the entrepreneur pledges the assets in the case of failure, then the financing condition becomes

$$p_H\left[R - \left(\frac{B}{\Delta p} + \xi c - A\right)\right] + (1 - p_H)(1-\xi)A \geqslant I.$$

Not pledging the asset in the case of failure facilitates financing if

$$p_H \xi c > [p_H + (1 - p_H)(1-\xi)]A,$$

which is never satisfied if $A > c$. Note that (1) the NPVs differ (the NPV is higher in the absence of pledging since the asset is then always maintained) and (2) more generally one should consider pledging only part of the asset.

Exercise 4.2 (diversification across heterogeneous activities). (i) Under specialization, the entrepreneur's net utility is (see Section 3.4)

$$U_b^i = \frac{\rho_1^i - 1}{1 - \rho_0^i} A \quad \text{for activity } i.$$

So, the entrepreneur prefers the low-NPV, low-agency-cost activity α if and only if

$$\frac{\rho_1^\alpha - 1}{1 - \rho_0^\alpha} > \frac{\rho_1^\beta - 1}{1 - \rho_0^\beta}. \tag{1}$$

(ii) Let R_2 denote the entrepreneur's reward if both activities succeed ($R_1 = R_0 = 0$). The entrepreneur must prefer behaving in both activities to misbehaving in both:

$$(p_H^2 - p_L^2) R_2 \geqslant B^\alpha I^\alpha + B^\beta I^\beta. \tag{2}$$

Now if the ratios I^α/I^β and B^α/B^β are sufficiently close to 1, a case we will focus on in the rest of the question, then the entrepreneur does not want to misbehave in a single activity either (the proof is similar to that in Section 4.2).

The entrepreneur solves

$$\max_{\{I^\alpha, I^\beta\}} \{(\rho_1^\alpha - 1)I^\alpha + (\rho_1^\beta - 1)I^\beta\}$$

s.t.

$$\rho_1^\alpha I^\alpha + \rho_1^\beta I^\beta - \frac{p_H^2}{p_H^2 - p_L^2}[B^\alpha I^\alpha + B^\beta I^\beta] \geqslant I^\alpha + I^\beta - A. \quad (3)$$

In contrast, the specialization solution solves the same program but with $p_H^2/[p_H^2 - p_L^2]$ replaced by $p_H/[p_H - p_L]$, which is bigger. Let

$$\tilde{\rho}_0^i \equiv p_H R^i - \frac{p_H^2}{p_H^2 - p_L^2} B^i > \rho_0^i.$$

Diversification reduces the agency cost. If

$$\frac{\rho_1^\alpha - 1}{1 - \tilde{\rho}_0^\alpha} < \frac{\rho_1^\beta - 1}{1 - \tilde{\rho}_0^\beta},$$

then the optimum is to have

$$I^\beta > I^\alpha.$$

But $I^\alpha = 0$ is not optimal. We need to reintroduce the incentive constraint according to which the entrepreneur does not want to shirk in activity β only (the one that yields the highest total private benefit): condition (2) (satisfied with equality so as to maximize borrowing capacity, and now labeled (2′)),

$$(p_H + p_L)(\Delta p)R_2 = B^\alpha I^\alpha + B^\beta I^\beta, \quad (2')$$

does not imply

$$p_H(\Delta p)R_2 \geqslant B^\beta I^\beta \quad (4)$$

if the ratio I^α/I^β is too small. Conditions (2′) and (4) (satisfied with equality) together define the optimal ratio I^α/I^β.

Exercise 4.4 ("value at risk" and benefits from diversification). Let R_0, R_1, and R_2 denote the entrepreneur's reward contingent on 0, 1, and 2 successes, respectively. The NPV (given that the entrepreneur will never receive rewards strictly above $\bar{R}$, we can reason on the risk-neutral zone in $u(\cdot)$ and use the NPV) is

$$2[p_H R - I].$$

To see whether the two projects can be financed simultaneously, minimize the nonpledgeable part of this NPV,

$$\tfrac{1}{4}[1+\alpha]R_2 + \tfrac{1}{2}[1-\alpha]R_1 + \tfrac{1}{4}[1+\alpha]R_0, \quad (1)$$

while providing incentives. To compute the entrepreneur's expected compensation above, note that the probability of two successes is

Pr(project 1 succeeds | work on project 1)

× Pr(project 2 succeeds | work on project 2 and success in project 1)

or $\frac{1}{2}[\frac{1}{2}(1+\alpha)]$. And so forth.

(i) The two incentive constraints are

$$\tfrac{1}{4}[1+\alpha]R_2 + \tfrac{1}{2}[1-\alpha]R_1 + \tfrac{1}{4}[1+\alpha]R_0 \geqslant 2B + R_0 \quad (2)$$

and

$$\tfrac{1}{4}[1+\alpha]R_2 + \tfrac{1}{2}[1-\alpha]R_1 + \tfrac{1}{4}[1+\alpha]R_0 \geqslant B + \tfrac{1}{2}R_1 + \tfrac{1}{2}R_0. \quad (3)$$

(ii) If $\bar{R}$ is large, one can then reward the entrepreneur only in the upper tail:

$$R_2 = \frac{8B}{1+\alpha}.$$

This value minimizes (1) subject to (2), and also satisfies (3).

(iii) When $\bar{R} < (8B)/(1+\alpha)$, the entrepreneur can no longer be rewarded solely in the upper tail to satisfy (2). Note that $R_0 = 0$ is optimal from (2) and (3). (2) can be satisfied by $\{R_2 = \bar{R},\ R_1 \leqslant \bar{R},\ R_0 = 0\}$ if and only if

$$\tfrac{1}{8}(3-\alpha)\bar{R} \geqslant B. \quad (4)$$

The question is then whether (3) is also satisfied.

- For *positive correlation* ($\alpha > 0$), increasing R_1 makes (3) harder to satisfy. Hence, minimizing the nonpledgeable income requires choosing the lowest R_1 that satisfies (2). This value satisfies (3) if and only if $B \geqslant (\frac{1}{2}R_1)$, or, after substitutions,

$$B \leqslant \tfrac{1}{4}\bar{R},$$

which is more constraining than (4).

- For *negative correlation* ($\alpha < 0$), increasing R_1 makes it easier to satisfy (3). While it is still optimal to set $R_2 = \bar{R}$, the binding constraint may now be (3) (and thus the nonpledgeable income exceeds $2B = 2p_H B/\Delta p$ here). Financing may be feasible even though it would not be so if project correlation were positive (but $\frac{1}{4}(1-\alpha)\bar{R}$ must exceed B).

Exercise 4.5 (liquidity of entrepreneur's claim). The entrepreneur's incentive constraint when the liquidity shock is observed by investors is

$$(1-\lambda)(\Delta p)R_b \geqslant B.$$

The NPV is

$$U_b = \text{NPV} = \lambda(\mu - 1)r_b + p_H R - I,$$

while the breakeven constraint is

$$\lambda(\mu_0 - 1)r_b + p_H R - (1-\lambda)p_H R_b \geqslant I - A.$$

As in the text, it is optimal to compensate the entrepreneur by providing her with liquidity (since $\mu > 1$) once R_b is equal to $B/(1-\lambda)\Delta p$. The level of liquidity, r_b^*, given to the entrepreneur is set by the breakeven constraint

$$\lambda(1-\mu_0)r_b^* + [I - A] = p_H\left(R - \frac{B}{\Delta p}\right).$$

It increases when more of the proceeds of reinvestment become pledgeable.

(ii) If λ is a choice variable, the entrepreneur faces multiple tasks. She solves

$$\max_{\{\lambda\in\{0,\bar{\lambda}\},\, p\in\{p_L,p_H\}\}} U_b(p,\lambda) = \{\lambda[\mu-\mu_0]r_b + (1-\lambda)pR_b - \lambda c + B\mathbf{1}_{\{p=p_L\}}\}.$$

The NPV is

$$U_b = \text{NPV} = \lambda(\mu - 1)r_b + p_H R - I - \lambda c.$$

For a given contract (R_b, r_b) the entrepreneur chooses

$$\lambda = \bar{\lambda} \quad \text{if } (\mu-\mu_0)r_b - pR_b \geqslant c.$$

Note that, for $p = p_H$, the entrepreneur does not "oversearch" for new investment opportunities as long as

$$(\mu-\mu_0)r_b - p_H R_b \leqslant (\mu - 1)r_b \iff (1-\mu_0)r_b \leqslant p_H R_b.$$

Suppose that one wants to implement $p = p_H$. Then

- either $\lambda = 0$, and then the outcome is the same as in the absence of a liquidity shock;
- or, more interestingly, $\lambda = \bar{\lambda}$ (which implies *a fortiori* that $\lambda = \bar{\lambda}$ if the entrepreneur deviates and chooses $p = p_L$):

$$U_b(p_H, \bar{\lambda}) \geqslant U_b(p_L, \bar{\lambda}) \iff (1-\bar{\lambda})(\Delta p)R_b \geqslant B.$$

Furthermore,

$$U_b(p_H, \bar{\lambda}) \geqslant U_b(p_H, 0) \iff (\mu-\mu_0)r_b - p_H R_b \geqslant c.$$

Hence, $R_b = B/[(1-\bar{\lambda})(\Delta p)]$, and so an added constraint with respect to subquestion (i) is

$$(\mu-\mu_0)r_b \geqslant c + p_H\frac{B}{(1-\bar{\lambda})\Delta p}.$$

Exercise 4.6 (project size increase at an intermediate date). Consider first the entrepreneur's date-1 behavior when the size has been doubled. If the entrepreneur has worked on the initial project, and using the perfect correlation between the two projects, the incentive constraint is

$$p_H\mathcal{R}_b \geqslant p_L\mathcal{R}_b + B.$$

If she shirked on the first project, then it is optimal to shirk again.

The date-0 incentive constraint is then

$$(1-\lambda)p_H R_b + \lambda p_H \mathcal{R}_b \geqslant B + (1-\lambda)p_L R_b + \lambda[p_L\mathcal{R}_b + B].$$

To obtain the nonpledgeable income, minimize the left-hand side of the latter inequality subject to the incentive constraints, yielding

$$\mathcal{R}_b = \frac{B}{\Delta p} \quad \text{and} \quad R_b = \frac{B}{(1-\lambda)\Delta p}.$$

Thus the nonpledgeable income is

$$(1+\lambda)p_H\frac{B}{\Delta p}.$$

Exercise 4.7 (group lending and reputational capital). (i) By assumption,

$$p_H\left(R - \frac{B}{\Delta p}\right) < p_H\left(R - \frac{B}{(1+a)\Delta p}\right) < I - A.$$

Under individual borrowing, the pledgeable income is $p_H[R-(B/\Delta p)]$, and so individual borrowing is not feasible. Under group lending, let R_b denote the borrower's individual reward when both succeed. They get 0 when at least one of them fails. The idea is that a borrower is punished "twice" for her failure: she gets no reward and also suffers from the other borrower's not receiving a reward. The incentive constraint is then

$$p_H(\Delta p)[(1+a)R_b] \geqslant B, \qquad (\text{IC}_b)$$

yielding pledgeable income per borrower

$$\mathcal{P} = p_{\rm H}R - p_{\rm H}^2\Big[\min_{\{\rm IC_b\}} R_{\rm b}\Big] = p_{\rm H}\Big[R - \frac{B}{(1+a)\Delta p}\Big].$$

Hence, group lending is not feasible either.

(ii) If both players are altruistic with $a = \frac{1}{2}$, they both cooperate in the unique equilibrium of the "stage-2" game. They have payoff $\frac{3}{2}$, since they enjoy the monetary gain of the other agent. More precisely, the utilities in the stage-2 game are as follows:

		Agent 1	
		C	D
Agent 2	C	$\frac{3}{2}, \frac{3}{2}$	$-1, 1$
	D	$1, -1$	$-\frac{3}{2}, -\frac{3}{2}$

Cooperating is a dominant strategy ($\frac{3}{2} > 1$ and $-1 > -\frac{3}{2}$), and so both cooperate.

If both agents are selfish ($a = 0$), the payoffs given in the statement of the question are those of a standard prisoner's dilemma and both agents defect.

(iii) The structure of payoffs is such that the altruistic agent gets nothing in the second stage if she misbehaves in the first stage. Consider the incentive constraint facing altruistic agents:

$$p_{\rm H}(\Delta p)(1+a)R_{\rm b} + \tfrac{3}{2}\delta \geqslant B$$

with $a = \frac{1}{2}$. The pledgeable income per borrower is

$$p_{\rm H}\Big(R - \frac{2B}{3\Delta p} + \frac{\delta}{\Delta p}\Big).$$

The financing is secured if

$$p_{\rm H}\Big(R - \frac{2B}{3\Delta p} + \frac{\delta}{\Delta p}\Big) \geqslant I - A.$$

From this, the minimum discount factor to secure financing is

$$\delta_{\min} = \frac{\Delta p}{p_{\rm H}}(I-A) - (\Delta p)R + \tfrac{2}{3}B > 0,$$

by assumption. The intuition is that the altruistic agent behaves in order to separate herself from the selfish agent and to build a reputation for being altruistic. The term $\delta/\Delta p$ reflects the gain from reputation and can be interpreted as the borrower's "social collateral."

Exercise 4.9 (borrower-friendly bankruptcy court). (i) • Monetary returns, such as L and r, that are not subject to moral hazard (or adverse selection) are optimally pledged to investors if financing is a constraint. This increases the income that is returned to investors without creating bad incentives for the entrepreneur.

• The entrepreneur's incentive constraint is (for a given realization of r)

$$[p_{\rm H}(r) - p_{\rm L}(r)]R_{\rm b} \geqslant B \quad \text{or} \quad (\Delta p)R_{\rm b} \geqslant B.$$

Condition (1) in the statement of the question says that continuation always maximizes social (total) value. However, systematic continuation (continuation for all r) generates too little pledgeable income to permit financing (right-hand side of condition (2) in the statement); on the other hand, systematic liquidation would generate enough pledgeable income (left-hand side of (2)).

Financing requires liquidating inefficiently. Intuitively, there is then no point giving $R_{\rm b}(r) > B/\Delta p$ for some rs in the case of continuation. The difference serves no incentive purpose and can be used to boost pledgeable income, allowing for more frequent continuation (in other words, it is more efficient to compensate the management with continuation rather than with money as long as incentives are sufficient). (Note: to prove this, generalize the optimization program in subquestion (ii) to allow for a choice of $R_{\rm b}(r)$ for $r \geqslant r^*$.)

(ii) • The borrower solves

$$\max \mathrm{NPV} = \max_{\{r^*\}} \Big\{E[r] + \int_{r^*}^{\bar r} \rho_1(r)f(r)\,\mathrm{d}r + \int_0^{r^*} Lf(r)\,\mathrm{d}r\Big\}$$

s.t.

$$E[r] + \int_{r^*}^{\bar r} \rho_0(r)f(r)\,\mathrm{d}r + \int_0^{r^*} Lf(r)\,\mathrm{d}r \geqslant I - A.$$

Clearly, r^* is the lowest value that satisfies the breakeven constraint. Condition (2) in the statement of the question implies that $0 < r^* < \bar r$. And, of course, $L \geqslant \rho_0(r^*)$.

(iii) • With a short-term debt contract, $d = r^*$, the firm will be able to repay its debt and continue if $r \geqslant r^*$. If $r < r^*$, the lenders are entitled to use default to liquidate. The investors do not want to renegotiate since $L > \rho_0(r^*)$.

• $\mathrm{d}r^*/\mathrm{d}A < 0$. A lower amount of equity calls for more pledgeable income.

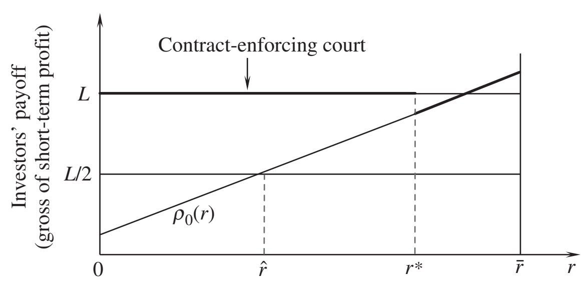

Figure 2

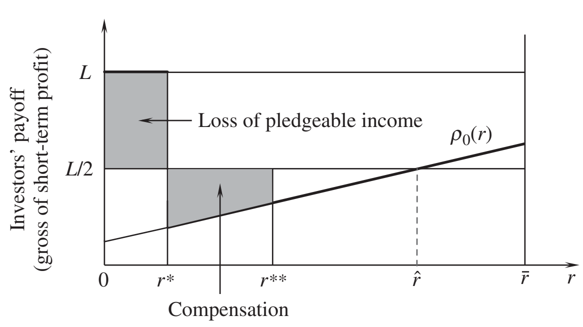

Figure 3

(iv) • Were the court to enforce the financial contract (previous questions), then the investors get (besides the short-term profit) L for $r < r^*$ and $\rho_0(r)$ for $r \geqslant r^*$. If $r^* > \hat{r}$, then the borrower-friendly court uniformly (weakly) reduces the available income whatever the continuation policy, as shown in Figure 2. Hence, investors, who just broke even, lose money and financing is no longer feasible.

(v) • A decrease in the investors' payoff (besides the short-term profit) from L to $\frac{1}{2}L$ over $[0, r^*]$ must be compensated by an increase in bankruptcy (see Figure 3). So, no bankruptcy occurs on some interval $[0, r^{**}]$ with $r^* < r^{**} \leqslant \bar{r}$, if this permits financing at all.

From an *ex ante* viewpoint, the lenders are not hurt since they break even regardless of the bankruptcy regime. The borrower suffers from poor enforcement (a simple way to check this is to note that, with a contract-enforcing court, she could choose to return only $\frac{1}{2}L$ to investors in the case of liquidation). See Chapter 16 for an in-depth study of who are the losers and who are the winners when public policies are modified.

Exercise 4.10 (benefits from diversification with variable-investment projects). (i) The analysis follows the lines of Section 3.4. The incentive constraint on project i with size I^i is

$$(\Delta p)R_{\mathrm{b}}^i \geqslant BI^i,$$

where R_{b}^i is the entrepreneur's reward in the case of success in project i; and so the pledgeable income is $\rho_0 I^i$.

The entrepreneur allocates A^i to project i, where

$$A^1 + A^2 = A.$$

Her total utility is

$$U_{\mathrm{b}} = \sum_i [(\rho_1 - 1)I^i] = \sum_i \left[(\rho_1 - 1)\left(\frac{A^i}{1-\rho_0}\right)\right] = \frac{\rho_1 - 1}{1 - \rho_0}A.$$

It does not really matter how the entrepreneur allocates her wealth between the two projects. In particular, there is no benefit to having a second project.

(ii) As in the case of fixed-investment projects, it is optimal to reward the entrepreneur only if the two projects succeed ($R_2 > 0$, $R_1 = R_0 = 0$). The two incentive constraints are

$$p_{\mathrm{H}}^2 R_2 \geqslant p_{\mathrm{H}} p_{\mathrm{L}} R_2 + \max_{i \in \{1,2\}} \{BI^i\}$$

and

$$p_{\mathrm{H}}^2 R_2 \geqslant p_{\mathrm{L}}^2 R_2 + B(I^1 + I^2).$$

Let

$$I \equiv I^1 + I^2.$$

Then

$$U_{\mathrm{b}} = \mathrm{NPV} = \sum_i [p_{\mathrm{H}} R I^i - I^i] = (\rho_1 - 1)I$$

and the financing condition becomes

$$p_{\mathrm{H}} R I - p_{\mathrm{H}}^2 R_2 \geqslant I - A.$$

Thus, everything depends only on total investment I, except for the first incentive constraint. For a given I, this constraint is relaxed by taking

$$I^1 = I^2 = \tfrac{1}{2}I.$$

The rest of the analysis proceeds as in Section 4.2. The first incentive constraint is satisfied if the second is. And so

$$U_{\mathrm{b}} = \frac{\rho_1 - 1}{1 - \rho_0'}A.$$

Exercise 4.11 (optimal sale policy). (i) The entrepreneur maximizes NPV,

$$\int_{s^*}^{1} (sR) f(s)\,\mathrm{d}s + F(s^*)L,$$

subject to the investors' breakeven constraint:

$$\int_{s^*}^{1} s\left(R - \frac{B}{\Delta p}\right) f(s)\,\mathrm{d}s + F(s^*)L \geqslant I - A, \qquad (\mu)$$

where use is made of the fact that the proceeds L from the sale should go to investors in order to maximize pledgeable income. One finds

$$s^*\left[\frac{R + \mu(R - B/\Delta p)}{1 + \mu}\right] = L.$$

Note that $s^* R = L$ if financing is not a constraint (A large), and

$$s^*\left[R - \frac{B}{\Delta p}\right] < L.$$

The optimal s^* trades off maximizing NPV (which would call for $s^* = L/R$) and pleasing investors (which would lead to $s^* = L/[R - (B/\Delta p)]$).

(Showoffs: we have assumed that it is optimal to induce the entrepreneur to exert effort when the firm is not liquidated. A sufficient condition for this to be the case is

$$(s - \Delta p)R \leqslant \max\left\{L, \left(s - \frac{B}{\Delta p}\right)R\right\};$$

that is, the pledgeable income is always lowest under continuation and shirking. To see this, consider state-contingent probabilities $x(s)$ of continuation and working, $y(s)$ of continuation and shirking, and $z(s)$ of liquidation.

Solve

$$\max_{\{x(\cdot),y(\cdot),z(\cdot)\}} \left\{\int_{\underline{s}}^{\bar{s}} [x(s)(sR) + y(s)[(s - \Delta p)R] + z(s)L] f(s)\,\mathrm{d}s\right\}$$

s.t.

$$\int_{\underline{s}}^{\bar{s}} \left[x(s)\left[\left(s - \frac{B}{\Delta p}\right)R\right] + y(s)[(s - \Delta p)R] + z(s)L\right] f(s)\,\mathrm{d}s \geqslant I - A$$

and $x(s) + y(s) + z(s) = 1$ for all s.)

(ii) Endogenizing $R_{\mathrm{b}}(s) \geqslant B/\Delta p$ for $s \geqslant s^*$ (where the threshold may differ from the one obtained in (a)), the expression for the NPV is unchanged. The breakeven constraint becomes

$$\int_{s^*}^{1} s[R - R_{\mathrm{b}}(s)] f(s)\,\mathrm{d}s + F(s^*)L \geqslant I - A.$$

The derivative with respect to $R_{\mathrm{b}}(s)$ is negative and so

$$R_{\mathrm{b}}(s) = B/\Delta p \quad \text{as long as } \mu > 0.$$

(iii) It is optimal to sell if $s = s_1$. Let R_{b}^* ($> B/\Delta p$ from the assumption made) be defined by

$$s_2(R - R_{\mathrm{b}}^*) = I - A.$$

If

$$B_0 \leqslant s_2 R_{\mathrm{b}}^*,$$

then the "career concerns" incentives are sufficient to prevent first-stage moral hazard. The only possible issue is then renegotiation. That is, if $s_1[R - B/\Delta p] > L$, the two parties are tempted to renegotiate.

If in contrast

$$B_0 > s_2 R_{\mathrm{b}}^*,$$

then even in the absence of renegotiation, there is first-stage moral hazard. Financing becomes infeasible.

Exercise 4.12 (conflict of interest and division of labor). (i) The incentive constraints are

$$\begin{aligned} p_{\mathrm{H}} R_{\mathrm{b}} + (1 - p_{\mathrm{H}})\hat{R}_{\mathrm{b}} - c &\geqslant p_{\mathrm{L}} R_{\mathrm{b}} + (1 - p_{\mathrm{L}})\hat{R}_{\mathrm{b}} - c + B \\ &\qquad \text{(no shirking on project choice)} \\ &\geqslant p_{\mathrm{H}} R_{\mathrm{b}} \\ &\qquad \text{(no shirking on maintenance)} \\ &\geqslant p_{\mathrm{L}} R_{\mathrm{b}} + B \\ &\qquad \text{(no shirking on either dimension).} \end{aligned}$$

The first two constraints can be rewritten as

$$(\Delta p)(R_{\mathrm{b}} - \hat{R}_{\mathrm{b}}) \geqslant B \quad \text{and} \quad \hat{R}_{\mathrm{b}} \geqslant \frac{c}{1 - p_{\mathrm{H}}}.$$

The third,

$$(\Delta p)R_{\mathrm{b}} + (1 - p_{\mathrm{H}})\hat{R}_{\mathrm{b}} \geqslant B + c,$$

is guaranteed by the other two.

(ii) The nonpledgeable income is

$$\min_{\{\mathrm{IC}\}}\{p_{\mathrm{H}} R_{\mathrm{b}} + (1 - p_{\mathrm{H}})\hat{R}_{\mathrm{b}}\} = p_{\mathrm{H}}\frac{B}{\Delta p} + \frac{c}{1 - p_{\mathrm{H}}}.$$

The financing condition is

$$p_{\mathrm{H}} R + (1 - p_{\mathrm{H}})L - p_{\mathrm{H}}\frac{B}{\Delta p} - \frac{c}{1 - p_{\mathrm{H}}} \geqslant I - A.$$

(iii) The agent in charge of maintenance is given $\hat{R}_{\mathrm{b}}$ conditional on failure and proper maintenance, and 0 otherwise. Her incentive constraint is

$$(1-p_H)\hat{R}_b \geqslant c.$$

So when given $\hat{R}_b = c/(1-p_H)$, this agent exerts care in maintaining the asset and receives no rent.

The entrepreneur's incentive constraint then becomes

$$(\Delta p)R_b \geqslant B.$$

The nonpledgeable income is now

$$p_H \frac{B}{\Delta p} + (1-p_H)\hat{R}_b = p_H \frac{B}{\Delta p} + c.$$

For more on the division of labor when multiple tasks are in conflict, see Dewatripont and Tirole (1999) as well as Review Problem 9.[5]

Exercise 4.14 (diversification and correlation). (i) The two incentive constraints are

$$p_H^2 R_2 \geqslant p_L^2 R_2 + 2B \quad \text{and} \quad p_H^2 R_2 \geqslant p_H p_L R_2 + B.$$

The first constraint can be rewritten as

$$p_H^2 R_2 \geqslant \frac{2p_H^2 B}{(p_H + p_L)\Delta p}. \qquad \text{(IC)}$$

The second constraint is satisfied if the first is. The pledgeable income is

$$2p_H R - \min_{\{IC\}}\{p_H^2 R_2\},$$

hence the result.

(ii) The entrepreneur receives $p_H R_2$ by behaving on both projects. When misbehaving (either on one or the two projects), the entrepreneur receives expected income $p_L R_2$. And so she might as well misbehave in both. The incentive constraint is then

$$p_H R_2 \geqslant p_L R_2 + 2B. \qquad \text{(IC)}$$

And so the pledgeable income is

$$2p_H R - \min_{\{IC\}}\{p_H R_2\} = 2p_H R - 2p_H \frac{B}{\Delta p}.$$

This yields the financing condition.

(iii) The incentive constraints are

$$[xp_H + (1-x)p_H^2]R_2 \geqslant [xp_L + (1-x)p_L^2]R_2 + 2B$$

and

$$[xp_H + (1-x)p_H^2]R_2 \geqslant [xp_L + (1-x)p_L p_H]R_2 + B.$$

5. Dewatripont, M. and J. Tirole. 1999. Advocates. *Journal of Political Economy* 107:1–39.

The second turns out to be satisfied if the first is. The financing condition becomes

$$p_H\left[R - \left[\frac{1-(1-x)(1-p_H)}{1-(1-x)(1-p_L-p_H)}\right]\frac{B}{\Delta p}\right] \geqslant I - A.$$

Ex ante (before financing), $x = 0$ facilitates financing. *Ex post* (after the investors have committed their funds), the entrepreneur's payoff,

$$[xp_H + (1-x)p_H^2]R_2,$$

is increasing in x and so $x = 1$. Note that the NPV is independent of x:

$$U_b = NPV = 2[p_H R - I].$$

Exercise 4.15 (credit rationing and the bias towards less risky projects). (i) Note, first, that the incentive compatibility constraint is the same regardless of the choice of project specification: letting R_b denote the entrepreneur's reward in the case of success (as usual, there is no point rewarding the entrepreneur in the case of failure), the incentive compatibility constraints are

$$(p_H^s - p_L^s)R_b \geqslant B \iff (p_H^r - p_L^r)R_b \geqslant B \iff (\Delta p)R_b \geqslant B.$$

The pledgeable income is therefore

$$\mathcal{P}^s = xp_H^s\left(R - \frac{B}{\Delta p}\right) + (1-x)L^s$$

for the safe variant, and

$$\mathcal{P}^r = xp_H^r\left(R - \frac{B}{\Delta p}\right) + (1-x)L^r$$

for the risky one.

Because $\mathcal{P}^s > \mathcal{P}^r$, choosing the safe variant facilitates funding. Lastly, $\bar{A}$ is defined by

$$\mathcal{P}^r \equiv I - \bar{A}.$$

The NPV is otherwise the same for both variants. Hence, U_b is the same provided the project is funded.

(ii) The entrepreneur having discretion over the choice of projects adds an extra dimension of moral hazard. Providing her with "high-powered incentives" (R_b in the case of success, 0 in the case of failure) is ideal for encouraging good behavior in the case of continuation, but it also pushes the entrepreneur to take risks, as[6]

$$x p_{\mathrm{H}}^{\mathrm{r}} R_{\mathrm{b}} > x p_{\mathrm{H}}^{\mathrm{s}} R_{\mathrm{b}}.$$

More generally, any incentive scheme that addresses the *ex post* moral-hazard problem ($(\Delta p)(R_{\mathrm{b}}^{\mathrm{S}} - R_{\mathrm{b}}^{\mathrm{F}}) \geqslant B$) encourages the choice of the risky variant unless the entrepreneur receives a reward (only) when the collateral value is high (L^{s}). But such a reward further reduces pledgeable income and may jeopardize financing altogether when $A < \bar{A}$, but $\mathcal{P}^{\mathrm{s}} \geqslant I - A$.

Exercise 4.16 (fire sale externalities and total surplus-enhancing cartelizations). (i) The representative entrepreneur's borrowing capacity i is determined by the investors' breakeven condition:

$$[x\rho_0 + (1-x)P]i = i - A,$$

where

$$\rho_0 \equiv p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right)$$

is the pledgeable income per unit of investment in the absence of distress.

Because it is individually optimal to resell all assets when in distress, $J = (1-x)I$, and so

$$P = P((1-x)I).$$

Furthermore, in equilibrium $i = I$, and so

$$I = \frac{A}{1 - [x\rho_0 + (1-x)P((1-x)I)]}.$$

The representative firm's NPV (or utility) is

$$U_{\mathrm{b}} = [x\rho_1 + (1-x)P((1-x)I) - 1]I$$

for the value of I just obtained.

(ii) In the case of cartelization, specifying that at most $z < 1$ can be resold on the market, and so $J \equiv (1-x)zI$, these expressions become

$$I = \frac{A}{1 - [x\rho_0 + (1-x)zP((1-x)zI)]}$$

and

$$U_{\mathrm{b}} = [x\rho_1 + (1-x)zP((1-x)zI) - 1]I.$$

6. Note that the choice of the risky project is perfectly detected in the case of liquidation, since liquidation then yields only L^{r} instead of the (higher) level L^{s}. The entrepreneur is, however, protected by limited liability and therefore cannot be punished for the wrong choice of project. (For the reader interested in contract theory: if we endogenized limited liability through large risk aversion below 0, we would need to assume that the safe project yields the low liquidation value L^{r} at least with positive probability. Otherwise, the entrepreneur could be threatened with a negative income in the case of low liquidation value and there would be no moral hazard in the choice of project.)

Let

$$H(z, I) \equiv (1-x)zP((1-x)zI).$$

Then

$$\frac{\partial H}{\partial z} = (1-x)[P + JP'].$$

Hence, H decreases with z if and only if the elasticity of demand is greater than 1.

Let us check that an elasticity of demand greater than 1 is consistent with the stability condition (incidentally, the same reasoning applies to the more general case in which only a fraction z of the assets are put up for sale). Simple computations show that

$$\frac{\mathrm{d}i}{\mathrm{d}I} = \frac{(1-x)^2 i^2 P'}{A},$$

and that the conditions

$$\frac{\mathrm{d}i}{\mathrm{d}I} > -1 \quad \text{and} \quad P + JP' < 0$$

are consistent if and only if

$$1 > x\rho_0 + 2(1-x)P.$$

This latter condition is not guaranteed by the fact that investment is finite ($1 > x\rho_0 + (1-x)P$), but is satisfied when x is large enough.

When the elasticity of demand exceeds 1,

$$I = \frac{A}{1 - [x\rho_0 + H(z, I)]}$$

decreases with z, and

$$U_{\mathrm{b}} = [x\rho_1 + H(z, I) - 1]I$$

decreases with z for two reasons: both the NPV per unit of investment and the investment decrease.

Simple computations show that

$$[A - J^2 P']\,\mathrm{d}I = (1-x)I^2[P + JP']\,\mathrm{d}z,$$

and so $\mathrm{d}I/\mathrm{d}z < 0$.

(iii) Let $\hat{\rho}_1 \equiv x\rho_1 + (1-x)zP$. The change in total surplus is given by

$$\mathrm{d}(U_{\mathrm{b}} + S^{\mathrm{n}}) = [(1-x)[P\,\mathrm{d}z + z\,\mathrm{d}P]I + (\hat{\rho}_1 - 1)\,\mathrm{d}I] - (1-x)I\,\mathrm{d}P,$$

where the first term (in brackets) on the RHS measures the change in the entrepreneur's utility and the second term the change in buyer surplus. And so

$$\mathrm{d}(U_{\mathrm{b}} + S^{\mathrm{n}}) = (1-x)PI\,\mathrm{d}z + (\hat{\rho}_1 - 1)\,\mathrm{d}I.$$

The term $(1-x)PI\,\mathrm{d}z$ corresponds to a better utilization of distressed assets (which are valued P by the marginal buyer) when $\mathrm{d}z > 0$, while the second term (the original one from the point of view of welfare analysis) stands for the social surplus created by an increase in borrowing capacity (associated with $\mathrm{d}z < 0$).

The total surplus increases when z decreases as long as

$$\hat{\rho}_1 - 1 \geqslant \frac{1-\hat{\rho}_0-(1-x)^2z^2(A/(1-\hat{\rho}_0))P'}{\eta-1},$$

where $\hat{\rho}_0 \equiv x\rho_0 + (1-x)zP$ and $\eta \equiv -P'J/P$.

Note that $\hat{\rho}_1$ can be increased without bound (by increasing ρ_1 keeping ρ_0 constant, i.e., by increasing B for a given ρ_0) without altering any other variable. So for $\hat{\rho}_1$ sufficiently large, total surplus increases.

Exercise 4.17 (loan size and collateral requirements). When collateral is pledged only in the case of failure, the NPV (also equal to the entrepreneur's utility) is

$$U_\mathrm{b} = p_\mathrm{H}R(I) - I - (1-p_\mathrm{H})[C - \phi(C)].$$

The entrepreneur's incentive compatibility constraint can be written as

$$(\Delta p)[R_\mathrm{b} + C] \geqslant BI,$$

where R_b denotes the entrepreneur's reward in the case of success. The investors' breakeven constraint is

$$p_\mathrm{H}[R(I) - R_\mathrm{b}] + (1-p_\mathrm{H})\phi(C) \geqslant I - A,$$

or, if the incentive constraint is binding,

$$p_\mathrm{H}\left[R(I) - \frac{BI}{\Delta p} + C\right] + (1-p_\mathrm{H})\phi(C) \geqslant I - A.$$

Maximizing U_b with respect to I and C subject to this latter constraint yields

$$p_\mathrm{H}R'(I) - 1 = \frac{\mu}{1+\mu}\left(\frac{p_\mathrm{H}B}{\Delta p}\right)$$

and

$$\phi'(C) = \frac{1}{1+\mu} - \frac{\mu}{1+\mu}\,\frac{p_\mathrm{H}}{1-p_\mathrm{H}},$$

where μ is the shadow price of the investors' breakeven constraint. As the balance sheet deteriorates, μ increases, I decreases, and C increases. Borrowing increases if the agency cost decreases; the impact of A on net borrowing $(I - A)$ is more ambiguous.

Exercise 5.1 (long-term contract and loan commitment). (i) The entrepreneur wants to carry on both projects as often as possible as this maximizes NPV. The pledgeable income in a contract that pays $R_\mathrm{b} = B/p_\mathrm{H}\Delta p$ in the case of two successes and continues in the case of first success is

$$p_\mathrm{H}(p_\mathrm{H}R - I) + \left(p_\mathrm{H}R - I - p_\mathrm{H}\frac{B}{\Delta p}\right);$$

hence, if it is weakly larger than 0, then the investors break even and the second project is financed if the first one was successful. If it is strictly larger than 0, then with investors breaking even, the entrepreneur has some additional income; it is optimal to take it in the form of a stochastic loan commitment in period 1.

(ii) Intuitively, ξ weakly increases in R, p_H and decreases in B, I, and p_L (as long as p_L is not too large). The optimal ξ is such that

$$(p_\mathrm{H} + \xi(1-p_\mathrm{H}))\left(p_\mathrm{H}R - I - p_\mathrm{H}\frac{B}{\Delta p}\right) + \left(p_\mathrm{H}R - I - \left(p_\mathrm{H}\frac{B}{\Delta p} - (1-\xi)(\Delta p)p_\mathrm{H}\frac{B}{\Delta p}\right)\right) = 0$$

or $\xi = 1$ if the solution to the previous equation exceeds 1.

(iii) The contract is renegotiation proof. Indeed, either $p_\mathrm{H}R - I - p_\mathrm{H}B/\Delta p < 0$ and then the lenders will not invest in the second project unless obliged to, or $\xi = 1$ and then the borrower wants to carry on the second project.

(iv) The described sequence of short-term contracts is behaviorally equivalent to the optimal long-term contract from (i).

Exercise 5.2 (credit rationing, predation, and liquidity shocks). (i) The incentive constraint is

$$(\Delta p)R_\mathrm{b} \geqslant B_1.$$

Hence, expected pledgeable income is

$$\rho_0^1 = p_\mathrm{H}\left(R_1 - \frac{B_1}{\Delta p}\right).$$

The entrepreneur receives funding if and only if

$$\rho_0^1 \geqslant I_1 - A.$$

(ii) • The competitor preys if the entrepreneur waits until date 1 to secure funding for the date-1 investment.

• To prevent predation, the entrepreneur can (publicly) secure at date 0 a credit line equal to $(I_1-\rho_0^1-a)$, or else obtain a guarantee that the date-1 project will be funded.

• Such long-term contracts are not renegotiated because they are *ex post* efficient (social surplus is maximized if the date-1 project is undertaken, as $p_{\mathrm{H}}R_1 > I_1$).

(iii) • The condition implies that unconditional financing of the two projects and date-0 shirking cannot allow investors to break even.

• x^* is given by

$$(\Delta q)(1-x^*)\left(\frac{p_{\mathrm{H}}B_1}{\Delta p}\right) \geqslant B_0.$$

• Suppose that $\rho_0^1 > I_1$. In states of nature where the initial contract specifies that the date-1 project is not financed, investors can offer to finance the project. They and the entrepreneur then get an extra rent (for example, $\rho_0^1 - I_1$ and $p_{\mathrm{H}}B_1/\Delta p$ if the investors make a take-it-or-leave-it renegotiation offer).

(iv) Termination is no longer a threat under renegotiation. The only way to induce the entrepreneur to behave at date 0 and date 1 is to give her, in the case of success at date 1, $R_{\mathrm{b}} = B_1/\Delta p$ if profit is equal to a, and $\mathcal{R}_{\mathrm{b}} > R_{\mathrm{b}}$ if it is equal to A, such that

$$(\Delta q)p_{\mathrm{H}}(\mathcal{R}_{\mathrm{b}} - R_{\mathrm{b}}) \geqslant B_0.$$

This reduces the date-1 pledgeable income from ρ_0^1 to

$$\rho_0^1 - q_{\mathrm{H}}p_{\mathrm{H}}(\mathcal{R}_{\mathrm{b}} - R_{\mathrm{b}}) = \rho_0^1 - q_{\mathrm{H}}\frac{B_0}{\Delta q}.$$

The condition in the statement of the exercise then implies that funding cannot be secured at date 0.

Exercise 5.3 (asset maintenance and the soft budget constraint). (i) Assume that the financiers can commit not to renegotiate the initial contract. The optimal contract for the entrepreneur maximizes the NPV,

$$U_{\mathrm{b}} = \left\{\int_0^{\bar{L}}\left[F(\rho^*(L))\rho_1 - \int_0^{\rho^*(L)}\rho f(\rho)\,\mathrm{d}\rho - 1 + [1-F(\rho^*(L))]L\right]g(L)\,\mathrm{d}L\right\}I,$$

subject to the financing constraint,

$$\left\{\int_0^{\bar{L}}\left[F(\rho^*(L))\rho_0 - \int_0^{\rho^*(L)}\rho f(\rho)\,\mathrm{d}\rho + [1-F(\rho^*(L))]L - \Delta(L)\right]g(L)\,\mathrm{d}L\right\}I \geqslant I - A,$$

and the incentive compatibility constraint for maintenance,

$$\left\{\int_0^{\bar{L}}[F(\rho^*(L))(\rho_1-\rho_0)+\Delta(L)]\ell(L)g(L)\,\mathrm{d}L\right\}I \geqslant B_0 I,$$

where

$$\ell(L) \equiv \frac{g(L)-\tilde{g}(L)}{g(L)}$$

is the likelihood ratio, and $\rho_1 - \rho_0 \equiv B/\Delta p$.

Letting μ and ν denote the shadow prices of these two constraints, one gets the formulae in the statement of the question by differentiating with respect to $\rho^*(L)$ and $\Delta(L)$.

(ii) The function $\rho^*(\cdot)$ obtained under commitment has slope exceeding -1 (except for very large L, for which the slope is equal to -1). This slope can be positive or negative. The soft-budget-constraint problem arises when ρ is smaller than $\rho_0 - L$ (allowing for negative values of ρ), i.e., for L small.

Exercise 5.4 (long-term prospects and the soft budget constraint). Go through the same steps as in Exercise 5.3, replacing "ρ_1" by "$\rho_1 + R_{\mathrm{L}}$," "ρ_0" by "$\rho_0 + R_{\mathrm{L}}$," eliminating the liquidation values, and making the functions $\rho^*(\cdot)$ and $\Delta(\cdot)$ functions of R_{L} instead of L. One finds

$$\rho^*(R_{\mathrm{L}}) = R_{\mathrm{L}} + \frac{\rho_1 + \nu\rho_0}{1+\nu} + \frac{\mu(\rho_1-\rho_0)}{1+\nu}\,\ell(R_{\mathrm{L}})$$

and

$$\Delta^*(R_{\mathrm{L}}) = 0 \quad \text{if } \nu\ell(R_{\mathrm{L}}) < \nu$$

(and if $\Delta^*(R_{\mathrm{L}}) > 0$, then $\rho^*(R_{\mathrm{L}}) = \rho_1 + R_{\mathrm{L}}$).

Exercise 5.5 (liquidity needs and pricing of liquid assets). (i) The borrower's utility, conditional on receiving funds, is equal to the project's NPV. Letting $(x_{\mathrm{L}}, x_{\mathrm{H}}) \in \{0,1\}^2$ denote the probabilities of continuation in low- and high-liquidity shock states, we have

$$U_{\mathrm{b}} = (1-\lambda)(\rho_1-\rho_{\mathrm{L}})x_{\mathrm{L}} + \lambda(\rho_1-\rho_{\mathrm{H}})x_{\mathrm{H}} - (I-A) - (q-1)(\rho_{\mathrm{H}}-\rho_0)x_{\mathrm{H}}.$$

Funding is feasible if

$$(1-\lambda)(\rho_0-\rho_L)x_L+\lambda(\rho_0-\rho_H)x_H \geqslant I-A+(q-1)(\rho_H-\rho_0)x_H.$$

For, the borrower needs no liquidity in order to cover the low shock: because $\rho_0>\rho_L$, the investors are willing to let their claim be diluted in order to continue. In contrast, the borrower needs to hoard $(\rho_H-\rho_0)$ Treasury bonds if $x_H=1$, in order to make up the shortfall between the liquidity shock and what can be raised on the capital market by diluting existing claimholders.

Clearly, $x_L=1$ as this both raises the borrower's objective function and relaxes the financing constraint. In contrast, $x_H=1$ raises the objective function as long as $(q-1)(\rho_H-\rho_0)\leqslant\lambda(\rho_1-\rho_H)$ but reduces the pledgeable income. If condition (2) in the statement of the exercise is satisfied, then $x_H=1$ is indeed optimal. Otherwise $x_H=0$ is optimal given the financing constraint. (Note that, were we to allow $0\leqslant x_H\leqslant 1$, that is, randomized liquidation, an $x_H\in(0,1)$ could be optimal when condition (2) is violated.)

(ii) Suppose neither (2) nor (3) is binding. Then each firm hoards $(\rho_H-\rho_0)$ Treasury bonds. But then there is excess demand for Treasury bonds as $T<\rho_H-\rho_0$.

Next, note that, for λ small, condition (2) cannot bind. Hence, (3) must bind:

$$q-1=\lambda\frac{\rho_1-\rho_H}{\rho_H-\rho_0}.$$

(iii) The new asset yields no liquidity premium since it yields no income in the bad state, and so $q'=1-\lambda$.

Exercise 5.6 (continuous entrepreneurial effort; liquidity needs). (i) The entrepreneur chooses probability of success p such that

$$\max_p\{pR_b-\tfrac{1}{2}p^2\}.$$

Hence,

$$p=R_b.$$

The breakeven constraint is

$$p(R-R_b)=I-A \quad\text{or}\quad R_b(R-R_b)=I-A.$$

Note that this equation is satisfied for $R_b=\frac{1}{2}R$.

(ii) The investors' breakeven condition is

$$I-A+\int_0^{\rho^*}\rho f(\rho)\,d\rho=F(\rho^*)R_b(R-R_b).$$

The entrepreneur maximizes

$$F(\rho^*)R_b^2$$

subject to the breakeven condition.

Exercise 5.7 (decreasing returns to scale). (i) The optimal policy maximizes the entrepreneur's expected utility, which is equal to the NPV,

$$U_b=rI+F(\rho^*)p_HR(I)-\Big(\int_0^{\rho^*}\rho f(\rho)\,d\rho\Big)I-I,$$

subject to the investors' breakeven constraint,

$$rI+F(\rho^*)p_H\Big(R(I)-\frac{BI}{\Delta p}\Big) \geqslant I-A+\Big(\int_0^{\rho^*}\rho f(\rho)\,d\rho\Big)I. \qquad (\mathrm{IR_I})$$

Let us assume that this constraint is binding. Taking the first-order conditions with respect to I and ρ^*, we obtain, after some manipulations,

$$p_H\Big[R'(I)-\frac{R(I)}{I}\Big]=\frac{1-r-\int_0^{\rho^*}(\rho^*-\rho)f(\rho)\,d\rho}{F(\rho^*)}. \qquad (1)$$

(ii) The right-hand side of (1) is decreasing in the cutoff ρ^*. The left-hand side of (1) is decreasing in I. Thus ρ^* and I comove positively. From ($\mathrm{IR_I}$), when the balance sheet deteriorates (A decreases), both I and ρ^* decrease. This implies, in particular, that the firm issues more short-term debt.

Exercise 5.8 (multistage investment with interim accrual of information about prospects). (i) • Start with variant (a) (uncertainty about τ). The optimal contract specifies a cutoff τ^* above which the firm should reinvest I_1.

The NPV (also equal to the entrepreneur's utility under a competitive capital market) is, for a given τ^*,

$$U_b(\tau^*)=\int_{\tau^*}^{\bar\tau}[(p_H+\tau)R-I_1]f(\tau)\,d\tau-I_0.$$

As usual, the incentive constraint (in the case of continuation) requires a minimum stake R_b in the case of success for the entrepreneur. R_b must satisfy

$$(\Delta p)R_b\geqslant B.$$

So the pledgeable income

$$\mathcal{P}(\tau^*) = \int_{\tau^*}^{\bar{\tau}} \left[(p_H + \tau)\left(R - \frac{B}{\Delta p}\right) - I_1\right] f(\tau)\, d\tau.$$

Financing requires that

$$\mathcal{P}(\tau^*) \geqslant I_0 - A.$$

U_b and $\mathcal{P}$ are maximized at τ_1^* and τ_0^* such that

$$(p_H + \tau_1^*)R = I_1$$

and

$$(p_H + \tau_0^*)\left(R - \frac{B}{\Delta p}\right) = I_1,$$

respectively. The entrepreneur is more eager to continue than the investors.

If $\mathcal{P}(\tau_1^*) \geqslant I_0 - A$, then the firm has deep pockets and the first-best continuation threshold τ_1^* is consistent with financing. So $\mathcal{P}(\tau_1^*) \equiv I_0 - A_1$. Otherwise, continuation must be less frequent as A declines:

$$\mathcal{P}(\tau^*) = I_0 - A.$$

But at the level A_0 at which

$$\mathcal{P}(\tau_0^*) = I_0 - A_0,$$

there is no longer the possibility to increase pledgeable income at the expense of value. For $A < A_0$, financing cannot be secured.

• The analysis of variant (b) proceeds similarly, with

$$U_b(R^*) = \int_{R^*}^{\infty} [p_H R - I_1] g(R)\, dR - I_0,$$

$$\mathcal{P}(R^*) = \int_{R^*}^{\infty} \left[p_H\left(R - \frac{B}{\Delta p}\right) - I_1\right] g(R)\, dR - I_0,$$

$$p_H R_1^* = I_1,$$

$$p_H\left(R_0^* - \frac{B}{\Delta p}\right) = I_1.$$

(ii) For $A = A_0$, the entrepreneur must give the entire pledgeable income in order to secure funding. So, she only takes

$$R_b = \frac{B}{\Delta p}$$

in the case of continuation, and

$$\mathcal{R} = (p_H + \tau)\frac{B}{\Delta p} = \frac{B}{(\Delta p)R}y,$$

where $B/(\Delta p)R < 1$ in variant (a), and $\mathcal{R} = p_H B/\Delta p$ in variant (b).

Exercise 5.9 (the priority game: uncoordinated lending leads to a short-term bias). (i) The first-best allocation maximizes the NPV:

$$\max_{\{I_1\}}\{r - I_1 + [p + \tau(I_1)]R\},$$

yielding

$$\tau'(I_1^*)R = 1.$$

Note that $I_1^* < r$ by assumption, and so an amount $(r - I_1^*)$ can be distributed at date 1. The date-1 payouts, r_b and r_l to borrower and lenders, and the date-2, success-contingent payouts, R_b and R_l, must satisfy

$$r_b + r_l + I_1^* = r,$$

$$R_b + R_l = R,$$

$$I = r_l + [p + \tau(I_1^*)]R_l.$$

This yields one degree of freedom.

(ii) Suppose that the entrepreneur secretly proposes the following contract to a (representative) lender: the lender's short-term claim increases by δr_l in exchange for the transfer of his long-term claim to the entrepreneur (by assumption, the entrepreneur is not allowed to defraud other investors of their short- or long-term claims). The lender is willing to accept this deal as long as

$$\delta r_l \geqslant [p + \tau(I_1)](\delta R_l).$$

Deepening investment decreases:

$$\delta I_1 = -\delta r_l.$$

The entrepreneur's interim utility increases by

$$\begin{aligned}\delta U_b &= [\tau'(I_1)(-\delta r_l)]R_b + [p + \tau(I_1)](\delta R_b)\\ &= [-\tau'(I_1)R_b + 1](\delta r_l) > 0\end{aligned}$$

when $I_1 = I_1^*$, since $\tau'(I_1^*)R = 1$ and $R_b < R$.

Note that the incentive to sacrifice the long-term profitability by increasing short-term debt decreases as R_b increases. Thus, it is optimal for the borrower to hold the smallest possible short-term claim ($r_b = 0$) and the largest long-term claim consistent with the investors' breakeven constraint and the collusion-proof constraint:

$$I = r - I_1 + [p + \tau(I_1)](R - R_b)$$

and

$$\tau'(I_1)R_b = 1,$$

where $I_1 < I_1^*$.

Exercise 5.10 (liquidity and deepening investment). (i) Let R_{b} denote the entrepreneur's reward in the case of success (she optimally receives 0 in the case of failure). The incentive constraint, as usual, is

$$(\Delta p)R_{\mathrm{b}} \geqslant B.$$

The necessary and sufficient condition for financing is that the pledgeable income exceeds the investors' outlay:

$$p_{\mathrm{H}}\Big(R - \frac{B}{\Delta p}\Big) \geqslant I - A.$$

(ii) The incentive compatibility condition is not affected by a deepening investment:

$$[(p_{\mathrm{H}} + \tau) - (p_{\mathrm{L}} + \tau)]R_{\mathrm{b}} \geqslant B \iff (\Delta p)R_{\mathrm{b}} \geqslant B.$$

The investors' breakeven condition is

$$[F(\rho^*)(p_{\mathrm{H}} + \tau) + [1 - F(\rho^*)]p_{\mathrm{H}}](R - R_{\mathrm{b}}) \geqslant I - A + \int_0^{\rho^*} \rho f(\rho)\,\mathrm{d}\rho.$$

(iii) The NPV (or borrower's utility) is

$$U_{\mathrm{b}} \equiv [F(\rho^*)(p_{\mathrm{H}} + \tau) + [1 - F(\rho^*)]p_{\mathrm{H}}]R - I - \int_0^{\rho^*} \rho f(\rho)\,\mathrm{d}\rho.$$

This NPV is maximized at

$$\rho^* = \tau R = \hat{\rho}_1.$$

Because

$$R_{\mathrm{b}} \geqslant \frac{B}{\Delta p},$$

the first best is implementable only in Case 1, which follows.

Case 1:

$$[F(\hat{\rho}_1)(p_{\mathrm{H}} + \tau) + [1 - F(\hat{\rho}_1)]p_{\mathrm{H}}]\Big(R - \frac{B}{\Delta p}\Big) \geqslant I - A + \int_0^{\hat{\rho}_1} \rho f(\rho)\,\mathrm{d}\rho$$

$$\iff [1 + \mu F(\hat{\rho}_1)]\rho_0 \geqslant I - A + \int_0^{\hat{\rho}_1} \rho f(\rho)\,\mathrm{d}\rho.$$

Case 2: if

$$[1 + \mu F(\hat{\rho}_0)]\rho_0 < I - A + \int_0^{\hat{\rho}_0} \rho f(\rho)\,\mathrm{d}\rho,$$

financing is infeasible.

Case 3: in the intermediate case, ρ^* is given by

$$[1 + \mu F(\rho^*)]\rho_0 = I - A + \int_0^{\rho^*} \rho f(\rho)\,\mathrm{d}\rho.$$

(iv) Whenever $\rho^* > \hat{\rho}_0$ (which is the generic case, conditional on financing), the firm must hoard liquidity in order to avoid credit rationing at the intermediate stage. The investors' maximal return on the deepening investment, $\mu\rho_0$, is smaller than the total value, $\mu\rho_1$, of this reinvestment.

Exercise 5.11 (should debt contracts be indexed to output prices?). (i) For a given policy $\rho^*(P)$, the NPV is

$$U_{\mathrm{b}} = \bar{P}r + E[F(\rho^*(P))p_{\mathrm{H}}PR] - I - E\Big[\int_0^{\rho^*(P)} \rho f(\rho)\,\mathrm{d}\rho\Big],$$

where expectations are taken with respect to the random price P. The investors' breakeven constraint is

$$\bar{P}r + E\Big[F(\rho^*(P))\Big[p_{\mathrm{H}}\Big(PR - \frac{B}{\Delta p}\Big)\Big]\Big] \geqslant I - A + E\Big[\int_0^{\rho^*(P)} \rho f(\rho)\,\mathrm{d}\rho\Big].$$

Let μ denote the shadow price of the budget constraint (we assume that $\mu > 0$). Then, taking the derivative of the Lagrangian with respect to $\rho^*(P)$ yields

$$\rho^*(P) = p_{\mathrm{H}}PR - \Big(\frac{\mu}{1 + \mu}\Big)\frac{p_{\mathrm{H}}B}{\Delta p}.$$

(ii) To implement the optimal policy through a state-contingent debt $d(P)$, one must have

$$\rho^*(P) = [Pr - d(P)] + \Big[p_{\mathrm{H}}\Big(PR - \frac{B}{\Delta p}\Big)\Big]$$

or

$$d(P) = Pr - \ell_0,$$

where

$$\ell_0 \equiv \frac{1}{1 + \mu}\Big(p_{\mathrm{H}}\frac{B}{\Delta p}\Big).$$

Exercise 6.1 (privately known private benefit and market breakdown). (i) If the borrower's private benefit B were common knowledge, then, if financed, the borrower would receive R_{b} in the case of success, with

$$R_{\mathrm{b}} \geqslant \frac{B}{\Delta p},$$

so as to induce her to behave. The project would be funded if and only if the pledgeable income exceeded the investment cost:

$$p_{\mathrm{H}}\Big(R - \frac{B}{\Delta p}\Big) \geqslant I.$$

Suppose that the borrower offers a contract specifying that she will receive R_b in the case of success and 0 in the case of failure (offering to receive more than 0 in the case of failure would evidently raise suspicion, and can indeed be shown not to improve the borrower's welfare). There are three possible cases:

(a) $R_b \geqslant B_H/\Delta p$ induces the borrower to work regardless of her type, and thus creates an information insensitive security for the lenders, who obtain

$$p_H(R - R_b) - I \leqslant p_H\left(R - \frac{B_H}{\Delta p}\right) - I < 0$$

using (1). So, such high rewards for the borrower cannot attract financing.

(b) $R_b < B_L/\Delta p$ induces the borrower to shirk regardless of her type. The lenders' claim is again information insensitive, and from (2) fails to attract financing.

(c) $B_L/\Delta p \leqslant R_b < B_H/\Delta p$: suppose that, in equilibrium, the good borrower offers a contract with a reward in this range, and that this attracts financing.[7] A bad borrower must then "pool" and offer the same contract: if she were to offer a different contract, her type would be revealed to the capital market and her project would not be funded. Furthermore, she receives utility from the project being funded at least equal to that of a good borrower (she receives the same payoff conditional on working and a higher payoff conditional on shirking). So, she is better off pooling with the good borrower than not being funded.

We conclude that equilibrium is necessarily a pooling equilibrium. It either involves no funding at all or funding of both types. From the study of cases (a) and (b), we also know that, in the case of funding, the good type behaves and the bad one misbehaves.

(ii) A necessary condition for funding is thus that

$$[\alpha p_H + (1-\alpha)p_L](R - R_b) \geqslant I.$$

Since $R_b \geqslant B_L/\Delta p$, there cannot be any lending if

$$\alpha < \alpha^*,$$

where

$$[\alpha^* p_H + (1-\alpha^*)p_L]\left(R - \frac{B_L}{\Delta p}\right) = I.$$

Thus, if the proportion of good borrowers is smaller than $\alpha^* \in (0,1)$, there is no lending at all. Bad borrowers drive out good ones and the loan market breaks down.

Suppose, next, that the proportion of good borrowers is high: $\alpha > \alpha^*$. The borrower may now be able to receive financing. Suppose that the borrower, regardless of her type, offers to receive R_b^* in the case of success and 0 in the case of failure, where

$$[\alpha p_H + (1-\alpha)p_L](R - R_b^*) = I.$$

Because $\alpha > \alpha^*$, $R_b^* > B_L/\Delta p$ and so the good borrower behaves. The investors' breakeven condition is therefore satisfied. It is an equilibrium for both types to offer contract $\{R_b^*, 0\}$ and for the capital market to fund the project.[8]

(iii) • The pooling equilibrium (which exists whenever $\alpha \geqslant \alpha^*$) exhibits no market breakdown. Indeed, there is *more lending under adverse selection than under symmetric information.*[9]

• It involves an externality between the two types of borrower. The good type obtains reward

$$R_b^* = R - I/[\alpha p_H + (1-\alpha)p_L]$$

in the case of success below that, $R - I/p_H$, that she would obtain under symmetric information. The good type thus *cross-subsidizes* the bad type, who would not receive any funding under symmetric information.

• The project's NPV conditional on being funded falls from $p_H R - I$ to $[\alpha p_H + (1-\alpha)p_L]R - I$ due to asymmetric information. The *quality of lending* is thus affected by adverse selection.

Exercise 6.2 (more on pooling in credit markets). A loan agreement specifying reward R_b in the case of success, and 0 in the case of failure, induces a proportion $H(R_b \Delta p)$ of borrowers to behave. This proportion is endogenous and increases with R_b. Thus

7. The reasoning can easily be extended to allow mixed strategies by the borrower and the capital market.

8. A more formal analysis of equilibrium behavior and of the equilibrium set can be performed along the lines of Section 6.4. We prefer to stick to a rather informal presentation at this stage.

9. This result and the following two can be found, for example, in de Meza and Webb's (1987) early contribution on the topic. (De Meza, D. and D. Webb. 1987. Too much investment: a problem of asymmetric information. *Quarterly Journal of Economics* 102:281–292.)

the lender's expected profit is

$$U_l = H(R_b \Delta p) p_H (R - R_b) + (1 - H(R_b \Delta p)) p_L (R - R_b).$$

Because $p_H > p_L$ so with only high-quality types, the level of R_b that satisfies the breakeven constraint of lenders could be larger than R_b when they face distribution H of borrowers. Thus, there is an externality among different types of borrowers.

Under a uniform distribution on $[0, \bar{B}]$ and for $p_L = 0$, the level of R_b maximizing pledgeable income is given by

$$\begin{aligned} 0 &= h(R_b \Delta p) p_H (R - R_b) \Delta p \\ &\quad - h(R_b \Delta p) p_L (R - R_b) \Delta p \\ &\quad - H(R_b \Delta p) p_H - (1 - H(R_b \Delta p)) p_L \\ &= h(R_b \Delta p)(R - R_b)(\Delta p)^2 - H(R_b \Delta p) \Delta p - p_L \\ &= \frac{1}{\bar{B}}(R - R_b) p_H^2 - \frac{R_b}{\bar{B}} p_H^2 \end{aligned}$$

or

$$R_b = \tfrac{1}{2} R.$$

Thus the pledgeable income is

$$\mathcal{P}(R_b) = \frac{1}{\bar{B}} \frac{p_H^2}{4} R^2,$$

and is smaller than I for $\bar{B}$ large enough.

Exercise 6.3 (reputational capital). (i) In this one-period adverse-selection problem, the bad type is always more eager to go on with a project than the good type. Thus, we may only have a pooling equilibrium. The assumptions imply that if we induce the bad type to work, or if we do not induce the good type to work, then the pledgeable income will not cover investment expenses. So, the only chance to receive funding is to induce the good type to work and the bad type to shirk. Under this type of contract, the pledgeable income is

$$\begin{aligned} &[\alpha p_H + (1 - \alpha) p_L]\left(R - \frac{b}{\Delta p}\right) \\ &\qquad = (p_H - (1 - \alpha)\Delta p)\left(R - \frac{b}{\Delta p}\right). \end{aligned}$$

(ii) First, note that the good type always works in the first period as $b < A \Delta p_1$.

In a pooling equilibrium, the bad type would always work. But then, the updated belief on the probability of the good type would still be α in period 2, and from the first inequality of the last displayed set of inequalities, and the result in (i), the project would not be financed in period 2. But this implies that the bad type would be better off shirking in period 1. So there is no pooling equilibrium.

In a separating equilibrium, the bad type would not work in period 1. Then, after a success in period 1, the updated belief on the probability of the good type would be α_S, and conditional on success in period 1 the project would be financed in period 2 (by the last assumed inequality) and the payoff to the borrower in the case of success would be

$$R - \frac{I - A}{p_H - (1 - \alpha_S)\Delta p}.$$

That, however, means that the bad type strictly prefers to work in period 1. Thus, there is no separating equilibrium.

The semiseparating equilibrium requires that the bad type is indifferent between working and shirking in period 1, that is,

$$B = (\Delta p_1)\left[p_L\left(R - \frac{I - A}{p_H - (1 - \alpha_S')\Delta p}\right) + B\right].$$

This determines the updated belief α_S' on the probability of the good type conditional on success in period 1, and thus determines the probability of the bad type working in period 1.

Exercise 6.5 (asymmetric information about the value of assets in place and the negative stock price reaction to equity offerings with a continuum of types). (i) The investors receive R_l in the case of success and 0 in the case of failure. The entrepreneur therefore issues equity if and only if

$$(p + \tau)(R - R_l) \geqslant pR \quad \Longleftrightarrow \quad \tau R \geqslant (p + \tau) R_l$$

and so there indeed exists a cutoff $p^* \in [\underline{p}, \bar{p}]$ such that the entrepreneur issues equity if and only if $p \leqslant p^*$.

(ii) The investors' breakeven condition is therefore

$$[E[p \mid p \leqslant p^*] + \tau] R_l = I \quad \text{or} \quad R_l = \frac{I}{m^-(p^*) + \tau}.$$

If interior, the cutoff satisfies

$$\tau R = (p^* + \tau) R_l \quad \text{or} \quad \frac{\tau R}{I} = \frac{p^* + \tau}{m^-(p^*) + \tau}.$$

Note also that $p^* > \underline{p}$: if p^* were equal to $\underline{p}$, then $m^-(p^*) = p^*$ and so types $\underline{p}$ and just above would be strictly better off issuing equity. The condition

$(m^-)' \leqslant 1$ does not suffice to guarantee uniqueness, though. Uniqueness, however, prevails if $(m^-)'$ is bounded away from 1 (for example, $(m^-)' = \frac{1}{2}$ in the case of a uniform distribution) and if $\tau R/I$ is close to 1.

For $p^* = \bar{p}$, $m^-(p^*) = E[p]$ (the prior expectation). And so the condition stated in (ii) ensures that the cutoff is interior.

Finally, if there are multiple equilibria, the one with the highest p^* yields the lowest stigma for equity issues since

$$R_1 = \frac{I}{m^-(p^*) + \tau}$$

is then smallest among equilibria.

For a uniform density, the equilibrium is, as we noted, unique, and, if interior, is given by

$$[\tfrac{1}{2}(p^* + \underline{p}) + \tau]\tau R = (p^* + \tau)I.$$

(ii) Let us now look at the stock price reaction. The market value prior to the announcement of the equity issue is equal to total value (given that investors will break even on average):

$$\begin{aligned} V_0 &= E(p)R + F(p^*)[\tau R - I] \\ &= [F(p^*)m^-(p^*) + [1 - F(p^*)]m^+(p^*)]R \\ &\quad + F(p^*)[\tau R - I]. \end{aligned}$$

The *ex post* value of shares upon an announcement is

$$V_1 = [m^-(p^*) + \tau]R - I.$$

And so

$$\begin{aligned} V_0 - V_1 = {} & [1 - F(p^*)] \\ & \times [m^+(p^*)R - [[m^-(p^*) + \tau]R - I]]. \end{aligned}$$

In the case of an interior equilibrium,

$$\begin{aligned} V_0 - V_1 = {} & [1 - F(p^*)]R \\ & \times \left[m^+(p^*) - \frac{p^*}{p^* + \tau}(m^-(p^*) + \tau)\right]. \end{aligned}$$

But

$$\frac{m^+(p^*)}{p^*} > 1 > \frac{m^-(p^*) + \tau}{p^* + \tau}.$$

Hence,

$$V_0 - V_1 > 0.$$

(iv) Let

$$H(p^*, \tau) \equiv \frac{\tau R}{I}[m^-(p^*) + \tau] - [p^* + \tau].$$

At the Pareto-dominant, interior equilibrium,

$$H_{p^*} < 0$$

(where the subscript denotes a partial derivative). Furthermore, and using the fact that $H = 0$ at an equilibrium,

$$H_\tau = [m^-(p^*) + \tau]\frac{R}{I} + \frac{p^* - m^-(p^*)}{m^-(p^*) + \tau} > 0.$$

Hence, p^* increase with τ. So does the volume $[1 - F(p^*)]I$.

Exercise 6.6 (adverse selection and rating). (i) • Condition (1) means that the pledgeable income of a good (bad) borrower exceeds (is lower than) the investors' investment $I - A$. The pledgeable income is equal to the expected income, $p_H R$, minus the entrepreneur's incompressible share, $p_H b/\Delta p$ (or $p_H B/\Delta p$).

• To see that no lending occurs in equilibrium, note that the bad type (type B) always derives a (weakly) higher surplus from being financed than a good type (type b). Hence, contracts that provide financing to a good type will also provide financing to a bad one (pooling behavior).

Condition (1) implies that one cannot offer a breakeven contract that induces the bad type to work. So any breakeven contract must induce misbehavior by the bad type. But condition (2) in turn implies that pooling contracts with stakes for the borrower in the interval $[b/\Delta p, B/\Delta p)$ generate a loss for the investors.

(ii) • In a separating equilibrium the good type chooses x and then offers R_b, and the bad type, which is recognized, chooses $x = 0$ and, from condition (1), receives no funding. Were the bad type to mimic the good type, she would get funding with probability $1 - x$; for, either the signal reveals the type and then she gets no funding, or the signal reveals nothing and the investors still believe they face a good type (we here use the fact that the equilibrium is separating).

Letting R_b^G denote the good type's "full information" (with net capital $A - rx$) contract (given by $p_H(R - R_b^G) = I - A + rx$), it must be the case that the bad type does not want to mimic the good type and prefers to keep her capital A instead. That is,

$$A \geqslant (1 - x)[p_L R_b^G + B] + x(A - rx)$$

or

$$A \geqslant x(A - rx) + (1 - x)\left[p_{\mathrm{L}}\left(R - \frac{I - A + rx}{p_{\mathrm{H}}}\right) + B\right],$$

which yields the condition in the question. This condition is satisfied with equality at the separating equilibrium (see the chapter).

Exercise 6.7 (endogenous communication among lenders). (i) First, consider date 1. The assumption $[\alpha p + (1-\alpha)q]R - I + \delta[\alpha p + (1-\alpha)q](R - I) < 0$ implies that a foreign bank would not lend at date 1 even if it faced no competition at date 1 and it remained a monopoly at date 2 and hence could offer $R_{\mathrm{b}} = 0$ in either period (with probability $\alpha p + (1 - \alpha)q$, the borrower would be known to be successful at date 2).

Thus, only the local bank will lend at date 1. Furthermore, the condition

$$qR - I + \delta q(R - I) < 0$$

implies that it would not lend to a bad type even if it faced no competition in either period. Hence, the local bank lends only to the good type. It offers $R_{\mathrm{b}}^1 = 0$.

In the absence of information sharing, foreign banks do not know whether the borrower succeeded at date 1, and therefore at date 2 (they put probability p on the borrower's being successful at date 2).

Note that the foreign banks do not want to make offers to the local borrower at date 2: suppose that they offer $R_{\mathrm{b}} < R$. Either the borrower will succeed and then the local, incumbent bank will offer a bit more ($R_{\mathrm{b}} + \varepsilon$), or it will fail and then the incumbent will not bid. Hence, a foreign bank can win the contest for the local firm only if the latter will fail. Hence, they do not bid, and the incumbent bank bids $R_{\mathrm{b}}^2 = 0$ if the borrower is successful (and does not finance otherwise). The local bank's profit (and thus each bank's profit since banks do not make profits in foreign markets) is

$$\pi^{\mathrm{ns}} = \alpha[pR - I + \delta p(R - I)],$$

where "ns" means "no sharing."

The borrower's *ex ante* utility is

$$U_{\mathrm{b}}^{\mathrm{ns}} = 0.$$

Suppose now that banks share their information. They are then Bertrand competitors at date 2 and make no profit at that date. But the local bank still lends at date 1 if the borrower's type is p: the profits and utilities are

$$\pi^{\mathrm{s}} = \alpha[pR - I] \quad \text{and} \quad U_{\mathrm{b}}^{\mathrm{s}} = \delta\alpha p(R - I).$$

Hence, banks do not want to share their information.

(ii) Suppose now that α is endogenous. Then $C(\alpha)$ needs to be subtracted from the borrower's previous utility (which is now a gross utility) in order to obtain the net utility.

In the absence of information sharing, the borrower is held up by the local bank, and so

$$\alpha^{\mathrm{ns}} = \pi^{\mathrm{ns}} = U_{\mathrm{b}}^{\mathrm{ns}} = 0.$$

Under information sharing, the borrower's investment is given by

$$\max_{\alpha}\{\delta\alpha p(R - I) - C(\alpha)\},$$

and so, for an interior solution,

$$C'(\alpha^*) = \delta p(R - I).$$

Then

$$\pi^{\mathrm{s}} = \alpha^*[pR - I] > \pi^{\mathrm{ns}}$$

and

$$U_{\mathrm{b}}^{\mathrm{s}} = \delta\alpha^* p(R - I) - C(\alpha^*).$$

Exercise 6.8 (pecking order with variable investment). (i) The separating program is

$$\max_{\{R_{\mathrm{b}}^{\mathrm{S}}, R_{\mathrm{b}}^{\mathrm{F}}\}} \{p_{\mathrm{H}} R_{\mathrm{b}}^{\mathrm{S}} + (1 - p_{\mathrm{H}}) R_{\mathrm{b}}^{\mathrm{F}}\}$$

s.t.

$$[p_{\mathrm{H}}(R^{\mathrm{S}} I - R_{\mathrm{b}}^{\mathrm{S}}) + (1 - p_{\mathrm{H}})(R^{\mathrm{F}} I - R_{\mathrm{b}}^{\mathrm{F}})] \geqslant I - A, \quad (\mathrm{IR_l})$$

$$q_{\mathrm{H}} R_{\mathrm{b}}^{\mathrm{S}} + (1 - q_{\mathrm{H}}) R_{\mathrm{b}}^{\mathrm{F}} \leqslant \tilde{U}_{\mathrm{b}}^{\mathrm{SI}}, \quad (\mathrm{M})$$

$$(\Delta p)(R_{\mathrm{b}}^{\mathrm{S}} - R_{\mathrm{b}}^{\mathrm{F}}) \geqslant BI. \quad (\mathrm{IC_b})$$

Note that ($\mathrm{IC_b}$) implies that the bad borrower works if she mimics the good one.

(ii) The key observation is that the solution to the separating program satisfies

$$R_{\mathrm{b}}^{\mathrm{F}} = 0.$$

That is, the good borrower receives nothing in the case of failure. In particular, if $R^{\mathrm{F}} I$ stands for the salvage value of the leftover assets, this salvage value is entirely transferred to the investors in the case of failure.

The proof of this observation is instructive. Suppose that $R_b^F > 0$. Consider a small increase $\delta R_b^S > 0$ in the borrower's reward in the case of success and a small decrease $\delta R_b^F < 0$ in her reward in the case of failure such that

$$p_H(\delta R_b^S) + (1 - p_H)(\delta R_b^F) = 0.$$

This change alters neither the objective function nor the investors' profit from the good borrower (see (IR_l)), but it relaxes the moral-hazard constraint (IC_b), and interestingly the mimicking constraint[10] as well since $q_H < p_H$. In words, a good borrower, who has a higher probability of success, cares relatively more about her income in the case of success and relatively less about her income in the case of failure than a bad borrower.

(iii) Because the weak monotonic-profit assumption is satisfied, Proposition 6.2 in the supplementary section implies that the separating allocation is the unique perfect Bayesian equilibrium allocation if and only if prior beliefs lie below some threshold α^*.

Exercise 6.9 (herd behavior). Entrepreneur 1, who moves first, chooses his best project, regardless of the state of nature. The investors then attach probability of success

$$m = \alpha p + (1 - \alpha)q$$

to the project. They are willing to go along with compensation R_b^1 such that

$$m(R - R_b^1) = I.$$

Now consider entrepreneur 2. In the unfavorable environment, she has no choice but choosing the strategy that gives a probability of success. Suppose now that she herds with entrepreneur 1 in the favorable environment. Her overall probability of success when she selects the same strategy as entrepreneur 1 is

$$\theta p + (1 - \theta)r.$$

So let R_b^U and R_b^F denote the second entrepreneur's compensation in the case of success depending on whether the environment is unfavorable or favorable, respectively:

$$q(R - R_b^U) = I \quad \text{and} \quad [\theta p + (1 - \theta)r](R - R_b^F) = I.$$

Herding behavior requires that

$$rR_b^F \geqslant pR_b^U$$

or

$$r\left[R - \frac{I}{\theta p + (1 - \theta)r}\right] \geqslant p\left[R - \frac{I}{q}\right].$$

This condition requires in particular that, despite herding, the choice of the same strategy by both entrepreneurs is sufficiently good news about the environment ($\theta p + (1 - \theta)r > q$) and therefore brings about much better financing terms for entrepreneur 2. It is satisfied, for example, if the project is hardly creditworthy in the unfavorable environment ($qR \simeq I$) and r is not too small.

Exercise 6.10 (maturity structure). In this simple example the good borrower can costlessly separate from the bad one by not hoarding any liquidity (i.e., setting short-term debt $d = r$). Because $\rho_0^G > \rho$, the good borrower knows that she will be able to find sufficient funds by going to the capital market at date 1 and diluting existing external claims. By contrast, the project will be stopped at date 1 for the bad borrower in the absence of liquidity hoarding, which would not be the case if the borrower resorted to hoarded liquidity rather than to the capital market to meet the liquidity shock.

This example is very special but it conveys the basic intuition: going back to the capital market is less costly for a good borrower than for a bad one if information about the firm's quality accrues in between. What is special about the example is that signaling by not hoarding liquidity is costless to the good borrower. Suppose that the liquidity shock is random and may exceed ρ_0^G. Then we know from Chapter 5 that it is optimal for the good borrower to hoard liquidity under symmetric information. So, signaling may involve insufficient continuation in general.

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10. The mimicking constraint can be shown to be binding. If it were not binding, the solution to the separating program would be the good borrower's full information contract. The borrower would thus obtain reward $BI^G/\Delta p$ in the case of success, and 0 in the case of failure, where I^G is determined by the good borrower's symmetric-information debt capacity. But then

$$q_H R_b^S + (1 - q_H)R_b^F = q_H BI^G/\Delta p > \hat{U}_b^{SI} = q_H BI^B/\Delta p,$$

where I^B is determined by the bad borrower's symmetric-information debt capacity. Because under symmetric information a good borrower can borrow more than a bad one, $I^G > I^B \geqslant 0$, and so (M) must be binding after all.

Exercise 7.1 (competition and vertical integration). (i) • The project can be financed because there is enough pledgeable income from condition (1).

• Feasible contracts:

$$R_1^{\mathrm{F}} + \theta_1 M \geqslant I \quad \text{and} \quad (\Delta p)(1-\theta_1)M \geqslant B.$$

For example, the debt contract,

$$R_1^{\mathrm{F}} = R^{\mathrm{F}} \quad \text{and} \quad \theta_1 = (I - R^{\mathrm{F}})/M$$

(which amounts to a debt $\mathcal{D} = I$), is an optimal contract. To obtain it as the unique optimal contract, one could, for example, add variable investment.

(ii) • The entrepreneur obtains

$$U_{\mathrm{b}} = R^{\mathrm{F}} + M - I$$

under an exclusive contract with the supplier.

By contrast, the industry profit when the rival obtains the enabling technology is

$$2(R^{\mathrm{F}} + D - I) + K < R^{\mathrm{F}} + M - I$$

from condition (2) and the profit-destruction effect. Because neither the supplier's nor the rival's rent (which is 0 under exclusivity) can decrease, the entrepreneur cannot gain from nonexclusivity.

• The supplier will not find it profitable to supply the enabling technology to the rival if and only if

$$R_1^{\mathrm{F}} + \theta_1 M \geqslant R_1^{\mathrm{F}} + \theta_1 D + \left[R^{\mathrm{F}} + \left(D - \frac{B}{\Delta p}\right) - (I - K)\right] \quad (3)$$

or

$$\theta_1(M - D) \geqslant R^{\mathrm{F}} + \left(D - \frac{B}{\Delta p}\right) - (I - K).$$

The term in square brackets in (3) is the difference between the rival's pledgeable income and the extra investment cost $I - K$. The solution is thus to offer enough equity to the supplier. Note that the borrower can always achieve this while maintaining borrower incentives: $(\Delta p)(1-\theta_1)D \geqslant B$. (If the borrower chose effort *after* observing the supplier's action, the incentive constraint would become $(\Delta p)(1-\theta_1)M \geqslant B$.)

Remark. For some parameter values an optimal debt/equity mix might involve a larger expected payment for the supplier than the investment I, but that is not a problem as the entrepreneur may demand a lump-sum payment equal to the difference up front, thus leaving the supplier with no rent.

Exercise 7.2 (benefits from financial muscle in a competitive environment). (i) • If $\rho > \rho_0(R)$, then the entrepreneur will not be able to withstand the liquidity shock if it occurs. Hence, it needs a liquidity cushion, perhaps in the form of a credit line.

• The NPV is

$$(1-\lambda)[\rho_1(R)] + \lambda[\rho_1(R) - \rho]z - I,$$

where $z = 1$ if the firm withstands the liquidity shock, and $z = 0$ otherwise. Hence,

(a) $z = 0$ if $\rho \geqslant \rho_1(R)$;

(b) $z = 1$ if $\rho < \rho_1(R)$ and there is enough pledgeable income to "secure a credit line,"

$$\rho_0(R) \geqslant I - A + \lambda\rho$$

or

$$(1-\lambda)\rho_0(R) - (I - A) \geqslant \lambda[\rho - \rho_0(R)]; \quad (5)$$

(c) $z = 0$ if (5) is not satisfied and

$$(1-\lambda)\rho_0(R) \geqslant I - A;$$

(d) no investment takes place if

$$(1-\lambda)\rho_0(R) < I - A.$$

(ii) • *Simultaneous choices*: under simultaneous choices, there is no commitment effect. Condition (1) and question (i) imply that the incumbent does not want to withstand her liquidity shock regardless of the existence of the entrant. The left inequality in (2) then implies that the entrant has enough pledgeable income to obtain financing if the incumbent does not build financial muscle (and wants to be financed from (3)); while the right inequality prevents the incumbent from investing ($I - A > \rho_0(C) > (1-\lambda)\rho_0(C)$).

• *Sequential choices*: suppose now that the incumbent chooses her financial structure first. The analysis of the simultaneous choice case shows that the incumbent cannot obtain financing without financial muscle. By contrast, condition (2) shows that the incumbent deters entry if she commits to withstand her liquidity shock. Condition (4) then implies that the incumbent has enough pledgeable income in a monopoly situation even if she withstands the costly liquidity shock.

Exercise 7.3 (dealing with asset substitution). (i) • The liquidation value L_0 is fully pledgeable. By contrast, only $R - R_b$ is pledged in the case of success, where

$$p_H R_b \geqslant p_L R_b + B.$$

Hence, the left-hand side of (1) is the pledgeable income.

• With a competitive capital market, the entrepreneur's utility is the NPV:

$$U_b^* = (1-x)L_0 + x p_H R - I.$$

• Optimal contracts must satisfy

$$(1-x)(L_0 - r_b) + x p_H (R - R_b) = I - A,$$

with

$$R_b \geqslant B/\Delta p.$$

For $A = \bar{A}$ the optimal contract is necessarily a debt contract ($r_b = 0$).

(ii) • Interpretation of equation (2). The NPV is

$$(1-x)L + x[p_H + \tau(L)]R - I.$$

Hence, $L = L_0$ maximizes the NPV, which is then equal to U_b^*.

• Consider a "step-function" contract: in the case of liquidation, the entrepreneur receives

$$\begin{aligned} 0 \quad &\text{if } L < L_0, \\ r_b \quad &\text{if } L \geqslant L_0. \end{aligned}$$

Furthermore, the entrepreneur receives $R_b = B/\Delta p$ in the case of continuation and success (this value minimizes both the nonpledgeable income and the incentive to cut down on maintenance to raise future profit). With this incentive scheme, the entrepreneur's utility

$$(1-x)r_b(L) + x[p_H + \tau(L)]R_b$$

is maximized either at $L = L_0$ or at $L = 0$. One therefore needs

$$(1-x)r_b + x p_H \frac{B}{\Delta p} \geqslant x[p_H + \tau(0)]\frac{B}{\Delta p}.$$

The threshold for financing that does not encourage asset substitution is given by

$$I - A^* = (1-x)(L_0 - r_b) + x p_H\left(R - \frac{B}{\Delta p}\right),$$

where r_b is given by the first inequality satisfied with equality.

Exercise 7.4 (competition and preemption). Let us first compute the first date $t_l < t_0$ at which lenders are willing to finance an entrepreneur who will later on be a monopolist:

$$I - e^{-r(t_0 - t_l)}A = e^{-r(t_0 - t_l)} p_H\left(M - \frac{B}{\Delta p}\right).$$

Thus no financing is feasible before date t_l.

Next, compute the earliest date $t_b < t_0$ at which the entrepreneur prefers to invest (as a monopolist) rather than just consuming her endowment:

$$\text{NPV} = e^{-r(t_0 - t_b)} p_H M - I = 0,$$

where the NPV is computed from date t_b on.

The condition in the statement of the question,

$$p_H M \gtreqless p_H\left(M - \frac{B}{\Delta p}\right) + A,$$

is equivalent to

$$t_b \gtreqless t_l.$$

Note that $t_b < t_l$ if $A = 0$.

(a) If $t_b \geqslant t_l$, then the equilibrium involves rent equalization, as in Fudenberg and Tirole (1985)[11]. Only one entrepreneur invests, and this at date t_b. (See Fudenberg and Tirole (1985) for a more rigorous description of the strategies.) This entrepreneur does not enjoy any rent relative to the entrepreneur who does not invest.

(b) If $t_b < t_l$, then we are back to a situation similar to the static game. Entrepreneurs are unable to invest before t_l, even though, starting from t_b, they would like to preempt their rival. (Again, we refer to Fudenberg and Tirole (1985) for more details about this type of situation.)

Exercise 7.5 (benchmarking). (i) Let us write the NPV, the breakeven constraint, and the incentive constraint. First, the NPV accounts for deadweight losses due to negative incomes:

$$\begin{aligned} U_b &= \text{NPV} \\ &= \rho[p_H D - (1-p_H)\theta b_2] \\ &\qquad + (1-\rho)[p_H^2 D + p_H(1-p_H)(M - \theta b_1) \\ &\qquad\qquad - (1-p_H)^2 \theta b_2] - I. \end{aligned}$$

11. Fudenberg, D. and J. Tirole. 1985. Preemption and rent equalization in the adoption of new technology. *Review of Economic Studies* 52:383–401.

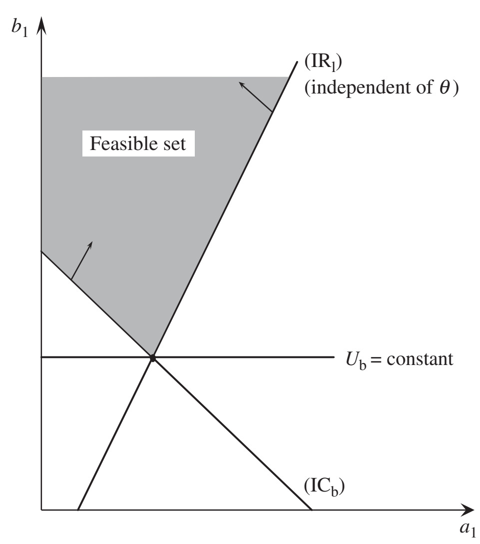

Figure 4

The breakeven constraint is

$$\rho[p_\mathrm{H}(D-a_2)+(1-p_\mathrm{H})b_2] + (1-\rho)[p_\mathrm{H}^2(D-a_2)+p_\mathrm{H}(1-p_\mathrm{H})(M-a_1+b_1) + (1-p_\mathrm{H})^2 b_2] \geqslant I-A. \tag{$\mathrm{IR_l}$}$$

Lastly, the incentive constraint is

$$\rho[a_2+(1+\theta)b_1] + (1-\rho)[p_\mathrm{H}[a_2+(1+\theta)b_1] + (1-p_\mathrm{H})[a_1+(1+\theta)b_2]] \geqslant \frac{B}{\Delta p}. \tag{$\mathrm{IC_b}$}$$

To show that one can set $a_2 = b_2 = 0$ without loss of generality, write the Lagrangian and the first-order condition. Equivalently, if $a_2 > 0$, we can decrease a_2 and increase a_1 so as to keep both $(\mathrm{IR_l})$ and $(\mathrm{IC_b})$ unchanged, and note that these two variables do not enter into the expression of the NPV; while, if $b_2 > 0$, we can decrease it and increase b_1 so that $(\mathrm{IR_l})$ and the NPV are kept intact, but $(\mathrm{IC_b})$ is then not binding.

The diagrammatic representation of the problem in the (a_1, b_1)-space is as in Figure 4.

(ii) • When ρ tends to 1: b_1 going to infinity has almost no cost in terms of NPV. Thus $(\mathrm{IC_b})$ becomes costless to satisfy, as in Section 7.1.1 in the case of perfect correlation.

• When θ goes to 0, then punishments are almost costless, and so again $(\mathrm{IC_b})$ can be satisfied without jeopardizing $(\mathrm{IR_l})$. Again there is basically no agency cost (as in the case in which firms have a large amount of collateral that the lenders value almost as much as the borrower).

Exercise 7.7 (optimal contracts in the Bolton–Scharfstein model). Consider a more general long-term contract in which the entrepreneur's reward contingent on different events is r_b^S if date-0 profit is D but there is no refinancing at date 1 (with probability z^S); and, if refinanced, R_b^SS (R_b^FS) when the entrepreneur succeeds in both periods (when she fails at date 0, but succeeds at date 1, respectively). When reinvesting at date 1, to "commit to" high effort, the entrepreneur should keep a high enough stake, i.e., R_b^SS and $R_\mathrm{b}^\mathrm{FS} \geqslant B/\Delta p$.

Fixing the continuation policy z^S and z^F, as long as the high effort is guaranteed the predation deterrence constraint is not affected by this enrichment of the contract space:

$$D \geqslant (z^\mathrm{S} - z^\mathrm{F})(M-D). \tag{PD}$$

The date-0 incentive compatibility constraint and investor's breakeven constraint, however, need to be modified:

$$\begin{aligned} z^\mathrm{S}R_\mathrm{b}^\mathrm{SS} + (1-z^\mathrm{S})r_\mathrm{b}^\mathrm{S} &\geqslant B_0 + p_\mathrm{L}[z^\mathrm{S}R_\mathrm{b}^\mathrm{SS} + (1-z^\mathrm{S})r_\mathrm{b}^\mathrm{S}] + (1-p_\mathrm{L})z^\mathrm{F}R_\mathrm{b}^\mathrm{FS} \\ \iff (\Delta p)[z^\mathrm{S}R_\mathrm{b}^\mathrm{SS} - z^\mathrm{F}R_\mathrm{b}^\mathrm{FS} + (1-z^\mathrm{S})r_\mathrm{b}^\mathrm{S}] &\geqslant B_0 \end{aligned} \tag{IC$'$}$$

and

$$\begin{aligned} I-A &\leqslant z^\mathrm{S}(D+D-R_\mathrm{b}^\mathrm{SS}-I) + (1-z^\mathrm{S})(D-r_\mathrm{b}^\mathrm{S}) \\ \iff I-A &\leqslant D + z^\mathrm{S}(D-I-R_\mathrm{b}^\mathrm{SS}) - (1-z^\mathrm{S})r_\mathrm{b}^\mathrm{S}. \end{aligned} \tag{IR$'$}$$

The entrepreneur's expected utility is

$$U_\mathrm{b} = z^\mathrm{S}R_\mathrm{b}^\mathrm{SS} + (1-z^\mathrm{S})r_\mathrm{b}^\mathrm{S} - A = \mathrm{NPV} = D - I + z^\mathrm{S}(D-I),$$

as usual, when (IR$'$) is binding.

As in Section 7.1.2, suppose (PD) is binding. (IC$'$) is binding; for, if it were not, z^F could be increased to relax (PD) without violating (IC$'$).

Then one can show that

- $R_\mathrm{b}^\mathrm{SS} \geqslant R_\mathrm{b}^\mathrm{FS}$ ($\geqslant B/\Delta p$): if $R_\mathrm{b}^\mathrm{SS} < R_\mathrm{b}^\mathrm{FS}$, then R_b^FS could be reduced so as to relax (IC$'$), which would contradict the fact that (IC′) is binding. And so $R_b^{FS} = B/\Delta p$.

- $r_b^S = 0$: suppose $z^S \in (0,1)$ and $r_b^S > 0$ (if $z^S = 1$, we could simply set $r_b^S = 0$). From (PD) being binding, the incentive constraint can be written as

$$z^S(R_b^{SS} - R_b^{FS}) + (1 - z^S)r_b^S + \frac{D}{M - D}R_b^{FS} = \frac{B_0}{\Delta p}.$$

Keeping z^S unchanged, we can decrease r_b^S and increase R_b^{SS} so that $z^S R_b^{SS} + (1 - z^S)r_b^S$ remains the same, i.e., in the case of date-0 success, one rewards the entrepreneur only in the case of continuation. There is no loss of generality in doing so since no constraint is affected, nor is the entrepreneur's objective function.

Exercise 7.8 (playing the soft-budget-constraint game vis-à-vis a customer). (i) At date 2, given success and in the absence of a date-1 contract, the customer would offer a purchasing price equal to 0 (or any arbitrarily small but positive amount) and the entrepreneur would accept. In this event, the entrepreneur and the investors get zero profit. Therefore, by playing wait-and-see, the customer would enjoy expected payoff $p_L v$, since the entrepreneur would shirk under this strategy. The same outcome prevails if the customer offers $R = 0$ at date 1.

Given that the entrepreneur has obtained funding at date 0, to induce a high probability of success at date 1 the customer needs to offer a price $R = R_l + B/\Delta p$. This is more profitable for the customer than offering a contract that is not incentive compatible:

$$p_H\left(v - R_l - \frac{B}{\Delta p}\right) > p_L v.$$

When this inequality holds, the NPV is

$$p_H\left(R_l + \frac{B}{\Delta p}\right) - I,$$

which is smaller than $(\Delta p)v - I$. On the other hand, if the condition above is violated, it is optimal for the customer to offer $R = 0$. But in this case the entrepreneur shirks and the project is not financed at date 0.

(ii) Suppose now that the entrepreneur issues short-term debt r_l at date 0. At time 1 the customer has to cover r_l in order for the firm to continue. It is as if date 1 were an initial financing stage at which the customer finances an investment with size r_l. The short-term debt can be chosen such that the customer refinances the project only if the entrepreneur works, i.e.,

$$p_L v < r_l.$$

Then, to induce the high effort, the customer offers a transfer price $R = B/\Delta p$, on top of r_l. The customer gets

$$p_H\left(v - \frac{B}{\Delta p}\right) - r_l.$$

By assumption $p_H(v - B/\Delta p) > p_L v$. It is possible to extract the full surplus from the customer by setting $r_l = p_H(v - B/\Delta p)$. This amount is greater than $I - A$ by assumption and so investors are willing to finance the project at date 0. The entrepreneur then gets

$$p_H\frac{B}{\Delta p} - A + [r_l - (I - A)],$$

which is equal to the NPV, $p_H v - I$. This is intuitive since both the initial investors and the customer get zero profit.

Exercise 7.9 (optimality of golden parachutes). Consider the following class of contract: when the entrepreneur reports a signal $s \in \{r, q\}$, the probability of continuation is z^s. She is paid R_b^s in the case of continuation and success, and T^s in the case of termination. In the latter event, the investors get $L_l^s = L - T^s \leqslant L$.

In the case of continuation, in order to overcome the moral-hazard problem, both R_b^r and R_b^q must exceed $B/\Delta p$. For the q-type entrepreneur, the (NM) constraint is now

$$z^r(q_H - \tau)R_b^r + (1 - z^r)T^r \leqslant z^q q_H R_b^q + (1 - z^q)T^q. \tag{NM′}$$

The investors' breakeven condition is

$$\begin{aligned}I - A \leqslant{}& \alpha[z^r r_H(R - R_b^r) + (1 - z^r)(L - T^r)] \\ &+ (1 - \alpha)[z^q q_H(R - R_b^q) + (1 - z^q)(L - T^q)]\end{aligned}$$

and the entrepreneur gets expected payoff

$$\begin{aligned}U_b &= \alpha[z^r r_H R_b^r + (1 - z^r)T^r] \\ &\quad + (1 - \alpha)[z^r q_H R_b^r + (1 - z^q)T^q] - A \\ &= \text{NPV} \\ &= \alpha[z^r r_H R + (1 - z^r)L] \\ &\quad + (1 - \alpha)[z^q q_H R + (1 - z^q)L] - I,\end{aligned}$$

under the investors' breakeven condition.

We claim that the following properties hold.

• (NM′) is binding. Otherwise, we could decrease either R_b^q or T^q and increase the pledgeable income unless $R_b^q = B/\Delta p$ and $T^q = 0$. But, in the latter case, from (NM′) being slack, we must have $z^q > 0$, then from $L > q_H(R - B/\Delta p)$ the pledgeable income can be increased by reducing z^q.

• $R_b^r = B/\Delta p$: if $R_b^r > B/\Delta p$, decreasing it boosts pledgeable income and relaxes (NM′).

• $T^r = 0$: suppose $T^r > 0$ and $z^r < 1$ (when $z^r = 1$, we can simply set $T^r = 0$). Following the logic of Section 7.2.1, a simultaneous change of T^r and z^r that keeps the pledgeable income constant must satisfy

$$\left[r_H\left(R - \frac{B}{\Delta p}\right) - L + T^r\right] \mathrm{d}z^r = (1 - z^r)\,\mathrm{d}T^r.$$

By doing so, the LHS of (NM′) changes by an amount equal to

$$\left[r_H\left(R - \frac{B}{\Delta p}\right) - L + (q_H - \tau)\frac{B}{\Delta p}\right] \mathrm{d}z^r;$$

(NM′) is relaxed by a simultaneous decrease of T^r and z^r. (If $z^r = 0$, we could instead decrease T^r to relax (NM′) and increase the pledgeable income.)

Incorporating these findings, the program becomes

$$\max\{\text{NPV} = \alpha[L + z^r(r_H R - L)] + (1-\alpha)[L + z^q(q_H R - L)] - I\}$$

$$\text{s.t.}\quad z^r(q_H - \tau)\frac{B}{\Delta p} = z^q q_H R_b^q + (1 - z^q)T^q, \quad (\text{NM}')$$

$$I - A = \mathcal{P} = \alpha\{L + z^r[r_H(R - B/\Delta p) - L]\} + (1-\alpha)\{L + z^q[q_H(R - R_b^q) - L] - (1 - z^q)T^q\}. \quad (\text{IR}')$$

• When $q_H R > L$, it is optimal not to adopt the golden parachute policy, $T^q = 0$: suppose $T^q > 0$. First, note that to satisfy (NM′) as an equality, $T^q < q_H B/\Delta p \leqslant q_H R_b^q$ as long as $\tau > 0$. Therefore, an increase in z^q relaxes (NM′) and increases the NPV. Consider a simultaneous change in z^q and T^q that leaves (NM′) unchanged:

$$(q_H R_b^q - T^q)\,\mathrm{d}z^q = -(1 - z^q)\,\mathrm{d}T^q.$$

Since $T^q < q_H R_b^q$, a decrease in T^q comes with an increase in z^q, which increases the NPV. This change is feasible since the pledgeable income is increased:

$$\mathrm{d}\mathcal{P} \approx [q_H(R - R_b^q) - L + T^q]\,\mathrm{d}z^q - (1 - z^q)\,\mathrm{d}T^q = (q_H R - L)\,\mathrm{d}z^q > 0.$$

• When $q_H R < L$, a golden parachute is optimal, $T^q > 0$ and $z^q = 0$. From $T^q < q_H R_b^q$, the relevant part in the pledgeable income can be written as

$$L + z^q[q_H R - L - (q_H R_b^q - T^q)] - T^q,$$

therefore decreasing z^q raises both the pledgeable income and the NPV. At the optimum $z^{q*} = 0$, and the optimal T^q is determined by (NM′):

$$T^{q*} = z^r(q_H - \tau)\frac{B}{\Delta p}.$$

It is also easy to check for both cases that the (NM) constraint of the r-type entrepreneur is not binding.

Exercise 7.10 (delaying income recognition). We look for a "pooling equilibrium" in which the entrepreneur keeps a low profile ($\hat{y}_1 = 0$) when successful ($y_1 = R_1$). To this end, let us compute the posterior probability α_{LB} (where "LB" stands for "late bloomer") that the entrepreneur has high ability at date 2 (H_2) following (reported) profit 1 at date 1 and (actual and reported) profit R_2 at date 2:

$$\alpha_{LB} = \Pr(H_2 \mid (0, R_2)) = \frac{A + B}{C + D},$$

where $A = \alpha\rho[r + r\tau]$, $B = (1-\alpha)(1-\rho)(r + q\tau)$, $C = \alpha[\rho r + (1-\rho)q + r\tau]$, and $D = (1-\alpha)[(1-\rho)r + \rho q + q\tau]$. The numerator represents the probability that the entrepreneur has ability H_2 and succeeds at date 2: with probability $\alpha\rho$, she had high ability at date 1 and still has high ability and so has average probability of success $r + r\tau$ (due to the date-1 hidden savings made when she is successful at date 1, which has probability r); with probability $(1-\alpha)(1-\rho)$ she had low ability at date 1 (and therefore had hidden savings with probability q) and became expert in the task (and so has probability of success $r + q\tau$). The denominator represents the total probability of date-2 success in this pooling equilibrium, and is computed in a similar way.

By contrast, the probability that the entrepreneur has type H_2 when she fails at date 2 is

$$\alpha_F = \frac{E + F}{G + H} < \alpha_{LB},$$

where $E = \alpha\rho[1 - (r + r\tau)]$, $F = (1-\alpha)(1-\rho)[1 - r - q\tau]$, $G = \alpha[1 - (\rho r + (1-\rho)q + r\tau)]$, and $H = +(1-\alpha)[1 - [(1-\rho)r + \rho q + q\tau]]$.

Suppose now that the entrepreneur reports $\hat{y}_1 = R_1$. Let

$$\alpha_{\mathrm{EB}} \equiv \Pr(H_2 \mid (R_1, R_2)) = \frac{I}{J+K}$$

(where $I = [\alpha\rho r + (1-\alpha)(1-\rho)q]r$, $J = [\alpha\rho r + (1-\alpha)(1-\rho)q]r$, and $K = [\alpha(1-\rho)r + (1-\alpha)\rho q]q$) and

$$\beta_{\mathrm{EB}} \equiv \Pr(H_2 \mid (R_1, 0)) = \frac{M}{N+O}$$

(where $M = [\alpha\rho r + (1-\alpha)(1-\rho)q](1-r)$, $N = [\alpha\rho r + (1-\alpha)(1-\rho)q](1-r)$, and $O = [\alpha(1-\rho)r + (1-\alpha)\rho q](1-q)$) denote the posterior beliefs when such an "early bloomer" (EB) succeeds and fails at date 2, respectively. It can be checked that a good report at date 1 improves one's reputation for an arbitrary date-2 performance,

$$\alpha_{\mathrm{EB}} > \alpha_{\mathrm{LB}} \quad \text{and} \quad \beta_{\mathrm{EB}} > \alpha_{\mathrm{F}},$$

and that

$$\alpha_{\mathrm{LB}} > \beta_{\mathrm{EB}}.$$

Intuitively, a late success is more telling than an early one if either the type has a reasonable probability to evolve or if an early success confirms what one already knows, namely, that the entrepreneur has high ability.

Now assume that

$$\alpha_{\mathrm{EB}} > \alpha_{\mathrm{LB}} > \hat{\alpha} > \beta_{\mathrm{EB}} > \alpha_{\mathrm{F}}.$$

Then, the entrepreneur keeps her job at date 3 if and only if she succeeds at date 2. Keeping a low profile at date 1 when $y_1 = R_1$ is then the optimal strategy because it increases the probability of date-2 success by τ.

Exercise 8.1 (early performance measurement boosts borrowing capacity in the variable-investment model). In the variable-investment model, the private benefit of shirking is BI, and the income in the case of success RI. Using the notation of Section 8.2.2, the incentive compatibility constraint is

$$(\sigma_{\mathrm{HH}} - \sigma_{\mathrm{LH}})R_{\mathrm{b}} \geqslant BI,$$

where R_{b} is the entrepreneur's reward in the case of success. The borrowing capacity is then given by the investors' breakeven constraint:

$$p_{\mathrm{H}}RI - \sigma_{\mathrm{HH}}\frac{BI}{\sigma_{\mathrm{HH}} - \sigma_{\mathrm{LH}}} = I - A.$$

And so

$$\begin{aligned} U_{\mathrm{b}} &= \sigma_{\mathrm{HH}}R_{\mathrm{b}} - A = (p_{\mathrm{H}}R - 1)I \\ &= \frac{\rho_1 - 1}{1 - (\rho_1 - \sigma_{\mathrm{HH}}B/(\sigma_{\mathrm{HH}} - \sigma_{\mathrm{LH}}))}A. \end{aligned}$$

In the absence of an intermediate signal, the expression is the same except that $\sigma_{\mathrm{HH}}/[\sigma_{\mathrm{HH}} - \sigma_{\mathrm{LH}}]$ is replaced by $p_{\mathrm{H}}/[p_{\mathrm{H}} - p_{\mathrm{L}}]$.

Exercise 8.2 (collusion between the designated monitor and the entrepreneur). When the signal is high, there is no collusion. In the absence of collusion, the entrepreneur obtains $\hat{R}_{\mathrm{b}}$ since it is in the interest of the monitor to exercise his options. Furthermore, the entrepreneur cannot receive more than $\hat{R}_{\mathrm{b}}$ from the assumption that the entrepreneur cannot receive income without being detected.

Suppose therefore that the signal is low. In the absence of collusion, the entrepreneur and the monitor both receive 0. Suppose that the entrepreneur instead offers to tunnel resources to the monitor. For a given choice of τ, the monitor agrees to collude if and only if his loss from exercising the options is compensated by the diverted resources:

$$s[p_{\mathrm{H}} - (\nu_{\mathrm{L}} - \tau)]R < T(\tau).$$

There is no collusion provided that

$$H(s) \equiv \max_{\{\tau\}}\{T(\tau) - s[p_{\mathrm{H}} - (\nu_{\mathrm{L}} - \tau)R]\} \leqslant 0.$$

Because $\partial H/\partial s < 0$, there is no collusion provided that s exceeds some threshold.

Exercise 9.1 (low-quality public debt versus bank debt). Consider the three possible financing options.

High-quality public debt. Such debt has probability p_{H} of being reimbursed. As usual, the incentive constraint is

$$(\Delta p)R_{\mathrm{b}} \geqslant B,$$

$$p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) \geqslant I - A,$$

and so such financing is doable only if

$$\Longrightarrow \quad A_3 = I - p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right).$$

The entrepreneur's utility is then the NPV:

$$U_{\mathrm{b}}^3 = p_{\mathrm{H}}R - I > 0.$$

Low-quality public debt. Such debt corresponds to the case in which the entrepreneur has too low a stake to behave; and this debt is repaid with probability p_{L}:

$$(\Delta p)R_{\mathrm{b}} < B \quad \text{and} \quad p_{\mathrm{L}}(R - R_{\mathrm{b}}) = I - A.$$

Hence,

$$A_1 = I - p_{\mathrm{L}}R.$$

The entrepreneur's utility is then

$$U_{\mathrm{b}}^1 = p_{\mathrm{L}}R + B - I > 0.$$

Monitoring. Follow the treatment in Chapter 9. To secure such financing with stake R_{m} for the monitor:

$$(\Delta p)R_{\mathrm{m}} \geqslant c \quad \text{and} \quad p_{\mathrm{H}}R_{\mathrm{m}} - c = I_{\mathrm{m}}.$$

And so a necessary and sufficient condition is

$$p_{\mathrm{H}}\left(R - \frac{b}{\Delta p}\right) - c \geqslant I - A,$$

yielding threshold

$$A_2 = I + c - p_{\mathrm{H}}\left(R - \frac{b}{\Delta p}\right),$$

and NPV

$$U_{\mathrm{b}}^2 = p_{\mathrm{H}}R - I - c.$$

Summing up, under the assumptions made in the statement of the exercise:

$$U_{\mathrm{b}}^3 > U_{\mathrm{b}}^2 > U_{\mathrm{b}}^1 > 0 \quad \text{and} \quad A_3 > A_2 > A_1.$$

So, financing is arranged as described in the statement of the question.

(A similar framework is used by Morrison[12], except that the monitor is risk averse (which makes it more costly to hire). Morrison allows the monitor to contract with a "protection seller" in the credit derivative market in order to pass the default risk on to this third party and to thereby obtain insurance. This reduces the monitor's incentive to monitor.)

Exercise 9.2 (start-up and venture capitalist exit strategy). (i) When the date-2 payoff can be verified at date 1, and there is no active monitor, the entrepreneur's reward, R_{b}, in the case of success must ensure incentive compatibility and allow investors to recoup their date-0 outlay:

$$(\Delta p)R_{\mathrm{b}} \geqslant B \quad \text{and} \quad p_{\mathrm{H}}(R - R_{\mathrm{b}}) \geqslant I - A.$$

Because

$$I - p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) > A,$$

these two conditions are mutually inconsistent.

Suppose, in contrast, that an active monitor receives R_{A} in the case of success. We now have two incentive compatibility conditions and one breakeven condition:

$$(\Delta p)R_{\mathrm{b}} \geqslant b,$$
$$(\Delta p)R_{\mathrm{A}} \geqslant c_{\mathrm{A}},$$

and

$$p_{\mathrm{H}}(R - R_{\mathrm{b}} - R_{\mathrm{A}}) \geqslant I - A.$$

Because

$$A > I - p_{\mathrm{H}}\left(R - \frac{b + c_{\mathrm{A}}}{\Delta p}\right),$$

these inequalities are consistent. The second and the third inequalities then bind, and so the NPV for the entrepreneur (which is equal to the total value created by the project minus the rent received by the monitor) is

$$p_{\mathrm{H}}R_{\mathrm{b}} - A = p_{\mathrm{H}}\left[R - \frac{c_{\mathrm{A}}}{\Delta p}\right] - I.$$

(ii) The conditions are

$$p_{\mathrm{H}}s[R - P] \geqslant c_{\mathrm{P}}$$

(the speculator makes money when he acquires information and exercises his call option in the case of good news),

$$(\Delta p)sP \geqslant c_{\mathrm{A}}$$

(this is the previous IC constraint with $R_{\mathrm{A}} = sP$), and

$$P \geqslant p_{\mathrm{H}}R$$

(the speculator cannot make money by refusing to monitor and purchasing the shares at price P).

Ignoring the last constraint yields the condition in the statement of the exercise. The third constraint requires that

$$\frac{c_{\mathrm{A}}}{c_{\mathrm{P}}} \geqslant \frac{1 - p_{\mathrm{H}}}{p_{\mathrm{H}}(\Delta p)}.$$

If this condition is not satisfied, the speculator does not have enough incentives to acquire the information when only the shares of the active monitor are

12. Morrison, A. 2002. Credit derivatives, disintermediation and investment decisions. Mimeo, Merton College, University of Oxford.

brought to the market at date 1. This means that the active monitor should be granted the right to "drag along" the shares (or some of the shares) of the limited partners in order to ensure the stock receives enough attention.

Exercise 9.3 (diversification of intermediaries). (i) Straightforward. Follows the lines of Chapters 3 and 4.

(ii) Similar to Chapter 4's treatment of diversification.

The venture capitalist obtains R_m if both projects succeed. The incentive constraints are

$$p_H^2 R_m \geqslant p_H p_L R_m + c \quad \text{(no shirking on monitoring one firm)}$$
$$\geqslant p_L^2 R_m + 2c \quad \text{(no shirking on monitoring both firms).}$$

As usual, it can be checked that only the latter constraint is binding. So

$$R_m \geqslant \frac{2c}{(\Delta p)(p_H + p_L)}.$$

The nonpledgeable income (aggregated over the two firms) is

$$2\left[p_H \frac{b}{\Delta p} + p_H\left(\frac{p_H}{p_H + p_L}\right)\frac{c}{\Delta p}\right].$$

Exercise 9.4 (the advising monitor model with capital scarcity). The entrepreneur's utility when enlisting a monitor is now equal to the NPV minus the rent derived by the monitor:

$$U_b^m = (p_H + q_H)\left(R - \frac{c}{\Delta q}\right) - I.$$

Note that U_b^m may no longer exceed

$$U_b^{nm} = p_H R - I,$$

even when $(\Delta q)R > c$.

Funding with a monitor on board is feasible if and only if

$$(p_H + q_H)\left(R - \frac{B}{\Delta p} - \frac{c}{\Delta q}\right) \geqslant I - A.$$

The presence of a monitor facilitates funding if and only if

$$(p_H + q_H)\left(R - \frac{B}{\Delta p} - \frac{c}{\Delta q}\right) > p_H\left(R - \frac{B}{\Delta p}\right)$$

or

$$q_H R > c + p_H \frac{c}{\Delta q} + q_H \frac{B}{\Delta p}.$$

The left-hand side is the increase in expected revenue; the right-hand side is the sum of the monitoring cost and the extra rents for the two agents.

Exercise 9.5 (random inspections). (i) Suppose first that the entrepreneur behaves with probability 1; then there is no gain from monitoring and so $y = 1$. But, in the absence of monitoring, the entrepreneur prefers to misbehave:

$$(\Delta p)R_b < B,$$

a contradiction. Conversely, suppose that the entrepreneur misbehaves with probability 1; because

$$\nu R_m > c,$$

the monitor monitors for certain ($y = 0$). But then the entrepreneur prefers to behave as

$$p_H R_b > 0.$$

Hence, the entrepreneur must randomize. For her to be indifferent between behaving and misbehaving, it must be the case that

$$p_H R_b = y(p_L R_b + B) + (1 - y) \cdot 0$$

or

$$y = \frac{p_H R_b}{p_L R_b + B}.$$

Similarly, the monitor must randomize. Indifference between monitoring and not monitoring implies that

$$(1 - x)p_H R_m + x(p_L + \nu)R_m - c = (1 - x)p_H R_m + x p_L R_m$$

or

$$x\nu R_m = c \quad \Longleftrightarrow \quad x = \frac{c}{\nu R_m}.$$

(ii) Assume that $p_H(R - B/\Delta p) < I - A$, so that financing is not feasible in the absence of a monitor. As usual, one should be careful here: because the monitor has no cash and thus cannot be asked to contribute to the investment and gets a rent, the borrower's utility differs from the NPV,

$$U_b = (1 - x)p_H R_b + x y(B + p_L R_b) - A$$
$$= p_H R_b - A,$$

using the indifference condition for the entrepreneur. The uninformed investors' breakeven condition is

$$\begin{aligned}\mathcal{P} \equiv\ & (1-x)p_{\mathrm{H}}(R-R_{\mathrm{b}}-R_{\mathrm{m}}) \\ & + x[yp_{\mathrm{L}}(R-R_{\mathrm{b}}-R_{\mathrm{m}}) \\ & \qquad + (1-y)(p_{\mathrm{L}}+v)(R-R_{\mathrm{m}})] \\ \geqslant\ & I-A.\end{aligned}$$

Note that $y = 0$ maximizes $\mathcal{P}$. First, if $x > 0$, a smaller y increases the amount of money returned to uninformed investors when correcting misbehavior. Second, it raises managerial discipline (reduces the level of R_{b} necessary to obtain incentive compatibility); indeed R_{b} can be taken equal to 0! (Note this would no longer hold if the entrepreneur could capture private benefit $b \in (0, B]$ before being fired.) The pledgeable income is then

$$\mathcal{P} = [(1-x)p_{\mathrm{H}} + x(p_{\mathrm{L}}+v)]\left[R - \frac{c}{xv}\right].$$

Noting that $\partial\mathcal{P}/\partial x > 0$ at $x = 0$ and $\partial\mathcal{P}/\partial x < 0$ at $x = 1$, the pledgeable income is maximized for x between 0 and 1. (The optimum does not, of course, involve $R_{\mathrm{b}} = 0$. We are just computing what it takes to obtain financing.)

(iii) We know from Chapter 8 that the entrepreneur is best rewarded on the basis of a sufficient statistic for her performance. Here, the monitor's information is not garbled by exogenous noise, unlike the final outcome. Hence, it would in principle be better to reward the management on the basis of information disclosed (in an incentive-compatible way) by the monitor. We leave it to the reader to derive the optimal contract when one allows the monitor to report on his observation of the entrepreneur's choice of effort.

Exercise 9.6 (monitor's junior claim). Let $R_{\mathrm{b}}^{\mathrm{S}}$ and $R_{\mathrm{b}}^{\mathrm{F}}$ denote the entrepreneur's rewards in the cases of success and failure. We are interested in situations in which the entrepreneur would choose the Bad project if left unmonitored:

$$(\Delta p)(R_{\mathrm{b}}^{\mathrm{S}} - R_{\mathrm{b}}^{\mathrm{F}}) < B.$$

Under monitoring, incentive compatibility requires that

$$(\Delta p)(R_{\mathrm{b}}^{\mathrm{S}} - R_{\mathrm{b}}^{\mathrm{F}}) \geqslant b,$$

where $\Delta p \equiv p_{\mathrm{H}} - p_{\mathrm{L}}$.

Similarly, the monitor's compensation scheme must satisfy

$$(\Delta p)(R_{\mathrm{m}}^{\mathrm{S}} - R_{\mathrm{m}}^{\mathrm{F}}) \geqslant c.$$

The uninformed investors are willing to lend if and only if

$$p_{\mathrm{H}}(R^{\mathrm{S}} - R_{\mathrm{b}}^{\mathrm{S}} - R_{\mathrm{m}}^{\mathrm{S}}) + (1-p_{\mathrm{H}})(R^{\mathrm{F}} - R_{\mathrm{b}}^{\mathrm{F}} - R_{\mathrm{m}}^{\mathrm{F}}) \geqslant I - A.$$

Finally, the borrower's utility is

$$p_{\mathrm{H}}R_{\mathrm{b}}^{\mathrm{S}} + (1-p_{\mathrm{H}})R_{\mathrm{b}}^{\mathrm{F}}.$$

It is therefore in the borrower's interest to minimize the monitor's rent,

$$p_{\mathrm{H}}R_{\mathrm{m}}^{\mathrm{S}} + (1-p_{\mathrm{H}})R_{\mathrm{m}}^{\mathrm{F}} - c,$$

subject to his incentive constraint,

$$(\Delta p)(R_{\mathrm{m}}^{\mathrm{S}} - R_{\mathrm{m}}^{\mathrm{F}}) \geqslant c.$$

This yields

$$R_{\mathrm{m}}^{\mathrm{F}} = 0 \quad \text{and} \quad R_{\mathrm{m}}^{\mathrm{S}} = \frac{c}{\Delta p}.$$

A necessary and sufficient condition for the borrower to have access to financing is

$$p_{\mathrm{H}}\left(R^{\mathrm{S}} - \frac{b+c}{\Delta p}\right) + (1-p_{\mathrm{H}})R^{\mathrm{F}} \geqslant I - A.$$

Exercise 9.7 (intertemporal recoupment). (i) *Long-term contracts.* The potential NPV is

$$V = 2p_{\mathrm{H}}R - (I_1 + I_2) - 2c.$$

Under *competition among monitors*, the borrower can obtain V, for example, by proposing a contract specifying that the selected monitor at date t, $t = 1, 2$, contributes I_{m}^t and receives R_{m}^t in the case of success (and 0 in the case of failure) such that

$$\begin{aligned} p_{\mathrm{H}}(R_{\mathrm{m}}^1 + R_{\mathrm{m}}^2) &= I_{\mathrm{m}}^1 + I_{\mathrm{m}}^2 + 2c, \\ (\Delta p)R_{\mathrm{m}}^t &= c. \end{aligned}$$

(The reader familiar with Sections 4.2 and 4.7 will notice that considering two incentive constraints, one per period, is in general not optimal. More on this later. However, we here show that the upper bound on the borrower's utility can be reached, and so we do not need to enter the finer analysis of "cross-pledging.")

Similarly, giving a stake R_{b}^t in the case of success (and 0 in the case of failure) such that

$$(\Delta p)R_{\mathrm{b}}^t \geqslant b$$

suffices (but is not necessary) to ensure borrower incentive compatibility.

Uninformed investors are then willing to finance the rest of the investments provided that

$$\sum_{t=1}^{2} p_{\mathrm{H}}[R - R_{\mathrm{b}}^{t} - R_{\mathrm{m}}^{t}] \geqslant \sum_{t=1}^{2} [I_t - I_{\mathrm{m}}^{t}]$$

or

$$p_{\mathrm{H}}[2R - R_{\mathrm{b}}^{1} - R_{\mathrm{b}}^{2}] \geqslant I_1 + I_2 + 2c.$$

The second condition in the statement of the exercise ensures that this condition can be met while satisfying the entrepreneur's incentive compatibility.

Under *monopoly in monitoring*, the same reasoning applies, with a few twists. First, the entrepreneur is rewarded only in the case of two successes. From Chapter 4, we know that she then gets R_{b} such that

$$[(p_{\mathrm{H}})^2 - (p_{\mathrm{L}})^2]R_{\mathrm{b}} \geqslant 2b.$$

(Two remarks. First, we do not allow termination to be used as a disciplining device. It is not renegotiation-proof anyway. Second, one can check that the monitor's incentive scheme can be designed so as to induce monitoring in both periods.) Second, the monitor then receives the NPV minus the entrepreneur's rent, i.e.,

$$V - \frac{(p_{\mathrm{H}})^2}{(p_{\mathrm{H}})^2 - (p_{\mathrm{L}})^2}\, 2b = V - 2\left(\frac{p_{\mathrm{H}}}{p_{\mathrm{H}} + p_{\mathrm{L}}}\right)\left(\frac{p_{\mathrm{H}} b}{\Delta p}\right).$$

(ii) *Short-term contracts.* Under competition, each monitor obtains no profit at date 2. The condition

$$I_1 + c > p_{\mathrm{H}}\left(R - \frac{b}{\Delta p}\right)$$

implies that no lending is feasible at date 1.

Under *monopoly*, the monitor will secure

$$p_{\mathrm{H}}\left(R - \frac{b}{\Delta p}\right) - I_2 - c > 0$$

at date 2, if he helps the firm obtain funding at date 1. His intertemporal profit is then

$$2p_{\mathrm{H}}\left(R - \frac{b}{\Delta p}\right) - (I_1 + I_2) - 2c > 0$$

(which is smaller than that under commitment because of the absence of cross-pledging across periods).

Exercise 10.1 (security design as a disciplining device). (i) R_{b}^{*} is the maximal entrepreneurial stake in the firm's payoff in the case of continuation that is consistent with the investors' breaking even. The entire short-term income (r in the case of success and L in the case of failure) is pledged to investors, and the project continues only in the case of date-1 success. The three conditions say that if the entrepreneur is rewarded R_{b}^{*} in the case of date-2 success, then

- $R_{\mathrm{b}}^{*} \geqslant B/\Delta p$: her date-2 incentive compatibility constraint is satisfied;
- $p_{\mathrm{H}}(R - R_{\mathrm{b}}^{*}) > L$: interference reduces the investors' income; and
- $(p_{\mathrm{H}}^{1} - p_{\mathrm{L}}^{1})[p_{\mathrm{H}} R_{\mathrm{b}}^{*}] \geqslant B_0$: the entrepreneur's date-1 incentive compatibility constraint is also satisfied.

(ii) From the definition of R_{b}^{*}, the project is financed, and from the three conditions, high efforts in both periods are guaranteed. Although there is an efficiency loss in terminating the project in the case of date-1 failure, this relaxes the date-1 incentive constraint and is optimal if p_{H}^{1} is large enough, that is, if the probability of interference is low enough.

The incentive scheme offered to the entrepreneur is that she is rewarded R_{b}^{*} if and only if she is successful in both periods; and the project is terminated if the date-1 income is equal to 0.

To implement this incentive scheme, the entrepreneur can issue two kinds of securities with different cash flow and control rights:

- short-term debt $d \in (0, \min\{L/p_{\mathrm{H}}, r\})$; debtholders receive control if d is not repaid at date 1; and
- long-term equities associated with control at time 1 if d is paid, and the following cash-flow rights: at date 1 equityholders receive the residual revenue ($r - d$ in the case of a date-1 success, and $\max\{0, L - d\}$ in the case of a date-1 failure); at date 2 they receive $R - R_{\mathrm{b}}^{*}$ in the case of success.

Debtholders interfere and terminate the project if there is no date-1 income, since

$$p_{\mathrm{H}} d < \min\{L, d\}.$$

Equityholders, when in control, do not interfere and so the project continues.

(iii) Suppose $R_{\mathrm{b}}^{*} = B/\Delta p$, and all three conditions still hold. Now if the entrepreneur is also paid

$r_b \in (0, r]$ in the case of date-1 success, the date-1 incentive constraint is relaxed:

$$(p_H^1 - p_L^1)[r_b + p_H R_b^*] \geqslant B_0.$$

But given that it is satisfied for $r_b = 0$, there is no benefit to boosting incentives in this way. Indeed, a positive r_b reduces the pledgeable income. The breakeven constraint of investors becomes more stringent:

$$I - A \leqslant p_H^1(r - r_b) + (1 - p_H^1)L + p_H^1[p_H(R - R_b^*)].$$

A positive r_b is not optimal as it makes the financing more difficult to arrange but has no incentive effect.

In general, a short-term bonus reduces the pledgeable income, while incentives are best provided by vesting the manager's compensation.

Exercise 10.2 (allocation of control and liquidation policy). (i) As usual, if financing is a binding constraint it is optimal to give 0 to the entrepreneur in the case of failure and to allocate the entire liquidation value L to investors in the case of liquidation. This increases the pledgeable income without perverse incentive effects or destruction of value. The entrepreneur maximizes her expected utility,

$$U_b = E_\omega[x(L, U_b^0)p_H R_b + [1 - x(L, U_b^0)]U_b^0],$$

subject to the incentive constraint,

$$(\Delta p)R_b \geqslant B,$$

and the investors' breakeven constraint,

$$E_\omega[x(L, U_b^0)p_H(R - R_b) + [1 - x(L, U_b^0)]L] \geqslant I - A.$$

The interesting case is when both the incentive and the participation constraints are binding. Let us rewrite the program as

$$\max E_\omega[x(L, U_b^0)(\rho_1 - \rho_0) + [1 - x(L, U_b^0)]U_b^0]$$
$$\text{s.t.}$$
$$E_\omega[x(L, U_b^0)\rho_0 + [1 - x(L, U_b^0)]L] = I - A.$$

Let $\mu \geqslant 1$ denote the multiplier of the participation constraint. We obtain

$$x^{SB}(\omega) = 1 \quad \text{if and only if } \rho_1 - U_b^0 \geqslant -(\mu - 1)\rho_0 + \mu L,$$

where "SB" stands for "second best."

As one would expect, continuation is less desirable when the liquidation value and the entrepreneur's alternative employment become more attractive (and, because of the difficulty of attracting financing, the liquidation value receives a higher weight than the entrepreneur's fallback option).

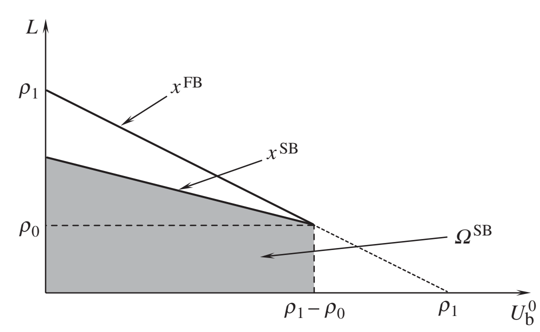

Figure 5

(ii) The first-best continuation rule is given by

$$x^{FB}(\omega) = 1 \quad \text{if and only if } \rho_1 - U_b^0 \geqslant L$$

(that is, $\mu = 1$). Ω^{SB} is included in Ω^{FB}, as described in Figure 5. More generally, Ω^{SB} shrinks as A decreases (μ increases).

(To show this, note that for $L < \rho_0$, everyone prefers to continue. So the interesting region is $L > \rho_0$.)

(iii) When the entrepreneur has control, the entrepreneur can guarantee himself $\rho_1 - \rho_0$ by choosing to continue. Second, renegotiation always leads to the first-best efficient outcome:

(a) *Continuation is first-best efficient.* If the initial contract makes the entrepreneur want to continue in the absence of renegotiation, there is nothing to renegotiate about (a necessary condition for renegotiation is the existence of gains from trade). If the entrepreneur prefers to liquidate (because of the existence of a golden parachute), the investors will want to compensate the entrepreneur to induce him to continue (the split of the gains from renegotiation depend on the relative bargaining powers).

(b) *Liquidation is first-best efficient.* Again, if the entrepreneur prefers to liquidate in the absence of renegotiation there is nothing to renegotiate about. Otherwise, the investors will "bribe" the entrepreneur to liquidate.

So

$$\Omega^{EN} = \Omega^{FB}.$$

Compare the investors' return with the pledgeable income derived in question (i). In Ω^{SB} and outside

Ω^{FB}, the decision rule is unchanged, and the investors cannot get more than ρ_0 and L, respectively. In $\Omega^{\text{FB}} - \Omega^{\text{SB}}$, the investors get at most ρ_0, while they were getting $L > \rho_0$. Thus, the project cannot be financed.

(iv) Under investor control, and in the absence of a golden parachute,

$$x^{\text{IN}}(\omega) = 1 \quad \text{if and only if } \rho_0 \geqslant L.$$

If $\rho_0 < L$, then investors cannot get more than under liquidation (there is no way the entrepreneur can compensate them). If $\rho_0 > L$, but $p_{\text{H}}R_{\text{l}} < L$, then the entrepreneur can offer a reduction of her stake in the case of success (while keeping $R_{\text{b}} \geqslant B/\Delta p$).

The project is financed since the investors get the same amount as in (i), except when $L > \rho_0$ and $\omega \in \Omega^{\text{SB}}$ for which they get more (L instead of ρ_0).

(v) In the absence of renegotiation, the investors liquidate if and only if

$$L - r_{\text{b}} \geqslant \rho_0.$$

The policy is renegotiated (toward liquidation) if

$$(\rho_1 - \rho_0) - U_{\text{b}}^0 \leqslant L - \rho_0 < r_{\text{b}}.$$

In contrast, if

$$(\rho_1 - \rho_0) - U_{\text{b}}^0 > L - \rho_0 > r_{\text{b}},$$

then there is no renegotiation and there is (inefficient relative to the first best) liquidation.

A *small* golden parachute increases the NPV while continuing to satisfy the financing constraint (an alternative would be to ask the investors to finance more than $I - A$ and let the entrepreneur save so as to be able to "bribe" the investors to induce continuation).

Exercise 10.3 (large minority blockholding). If $\xi < (\tau + \mu)s_2R$, then the large shareholder and the uninformed (majority) investors have aligned interests. The majority shareholders therefore always follow the large shareholder's recommendation.

Let us therefore assume that $\xi > (\tau + \mu)s_2R$. Let us look for an equilibrium in which the entrepreneur makes her suggestion "truthfully" (just announces her preferred modification). In state 2, the large shareholder seconds the entrepreneur's proposal. He makes a counterproposal in states 1 and 3.

The majority shareholders then go along with the joint proposal (in state 2). In the case of disagreement, the majority shareholders select the entrepreneur's proposal, that of the large shareholder, or the status quo so as to solve

$$\max\{-\beta\mu + \tau(1-\kappa), \beta\tau - \mu(1-\beta)(1-\kappa), 0\}.$$

Note that in the equilibrium under consideration both the entrepreneur and the minority blockholder have incentives to report their preferences truthfully (and that there are other equilibria where this is not the case).

Exercise 10.4 (monitoring by a large investor). Let

$$U_{\text{b}}(x) \equiv p_{\text{H}}R + [\xi + (1-\xi)x][\tau R - \gamma] - c_{\text{m}}(x) - I$$

denote the NPV (the NPV is equal to the borrower's utility because there is no scarcity of monitoring capital, and therefore no rent to be left to the monitor). Let

$$\mathcal{P}(x) \equiv [p_{\text{H}} + [\xi + (1-\xi)x]\tau]\left(R - \frac{B}{\Delta p}\right) - c_{\text{m}}(x)$$

denote the income that can be pledged to investors given that (a) the entrepreneur's stake must exceed $B/\Delta p$ in order to elicit good behavior, and (b) the monitor's expected income must compensate him for his monitoring cost. Concerning the last point, the monitor's reward R_{m} in the case of success and investment contribution I_{m} must satisfy the following breakeven and incentive conditions:

$$p_{\text{H}}R_{\text{m}} = I_{\text{m}} + c_{\text{m}}(x) \quad \text{and} \quad (1-\xi)\tau R_{\text{m}} = c'_{\text{m}}(x).$$

Note that

$$U_{\text{b}}(x) - \mathcal{P}(x) = [\xi + (1-\xi)x]\left(\tau\frac{B}{\Delta p} - \gamma\right) + \text{constant},$$

and so is decreasing in x.

If there is a shortage of pledgeable income, the optimal monitoring level given by (10.11) and maximizing the NPV,

$$c'_{\text{m}}(x^*) = (1-\xi)(\tau R - \gamma),$$

is no longer adequate. Indeed

$$U'_{\text{b}}(x^*) = 0 \quad \Longrightarrow \quad \mathcal{P}'(x^*) > 0.$$

Thus, the monitoring intensity must increase beyond x^*:

$$c'_{\text{m}}(x) > (1-\xi)(\tau R - \gamma).$$

If funding is feasible, then x is given by (the smallest value satisfying)

$$\mathcal{P}(x) = I - A.$$

Let $\hat{x}$ ($> x^*$) be defined by

$$c'_{\mathrm{m}}(\hat{x}) \equiv (1-\xi)\tau\left(R - \frac{B}{\Delta p}\right).$$

Because the pledgeable income no longer increases above $\hat{x}$, funding is feasible only if

$$\mathcal{P}(\hat{x}) \geqslant I - A.$$

Exercise 10.5 (when investor control makes financing more difficult to secure). (i) The incentive constraint is as usual

$$p_{\mathrm{H}}R_{\mathrm{b}} \geqslant p_{\mathrm{L}}R_{\mathrm{b}} + B, \tag{1}$$

yielding pledgeable income

$$\mathcal{P}_1 \equiv p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right).$$

The entrepreneur can receive funding if and only if

$$\mathcal{P}_1 \geqslant I - A.$$

(ii) Assume entrepreneur control. Either

$$\nu R_{\mathrm{b}} \leqslant \gamma,$$

and then the entrepreneur does not engage in damage control when shirking. The relevant incentive constraint remains (1), or

$$\nu R_{\mathrm{b}} > \gamma,$$

and the incentive constraint becomes

$$p_{\mathrm{H}}R_{\mathrm{b}} \geqslant (p_{\mathrm{L}} + \nu)R_{\mathrm{b}} + B - \gamma. \tag{2}$$

If

$$\nu\left(\frac{B}{\Delta p}\right) \leqslant \gamma,$$

then the incentive constraint is unchanged when $R_{\mathrm{b}} = B/\Delta p$, and so the pledgeable income (the maximal income that can be pledged to investors while preserving incentive compatibility) is still $\mathcal{P}_1$.

(iii) Under investor control, the damage-control action is selected, and so the incentive constraint becomes

$$p_{\mathrm{H}}R_{\mathrm{b}} - \gamma \geqslant (p_{\mathrm{L}} + \nu)R_{\mathrm{b}} + B - \gamma \tag{3}$$

or

$$(\Delta p - \nu)R_{\mathrm{b}} \geqslant B.$$

The new pledgeable income is

$$\mathcal{P}_2 = p_{\mathrm{H}}\left(R - \frac{B}{\Delta p - \nu}\right),$$

and is smaller than under entrepreneur control.

Exercise 10.6 (complementarity or substitutability between control and incentives). (i) As usual, this condition is

$$p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) \geqslant I - A.$$

(ii) Under *entrepreneur control*, the profit-enhancing action is not chosen in combination with the high effort since

$$(p_{\mathrm{H}} + \tau_{\mathrm{H}})R_{\mathrm{b}} - \gamma < p_{\mathrm{H}}R_{\mathrm{b}}$$

(since $\tau_{\mathrm{H}}R_{\mathrm{b}} < \tau_{\mathrm{H}}R < \gamma$).

Thus, to induce the high effort, R_{b} must satisfy $(\Delta p)R_{\mathrm{b}} \geqslant B$.

But then it is also optimal for the entrepreneur not to misbehave and choose the profit-enhancing action simultaneously:

$$\begin{aligned}(p_{\mathrm{L}} + \tau_{\mathrm{L}})R_{\mathrm{b}} + B - \gamma &\leqslant p_{\mathrm{H}}R_{\mathrm{b}} + \tau_{\mathrm{L}}R_{\mathrm{b}} - \gamma \\ &< p_{\mathrm{H}}R_{\mathrm{b}},\end{aligned}$$

since $R_{\mathrm{b}} < R$. The analysis is therefore the same as in (i).

Under *investor control*, it is a dominant strategy for the investors to select the profit-enhancing action. Hence, the manager's incentive constraint becomes

$$(p_{\mathrm{H}} + \tau_{\mathrm{H}})R_{\mathrm{b}} \geqslant (p_{\mathrm{L}} + \tau_{\mathrm{L}})R_{\mathrm{b}} + B$$

or

$$(\Delta p + \Delta\tau)R_{\mathrm{b}} \geqslant B.$$

The pledgeable income increases with investor control if and only if

$$(p_{\mathrm{H}} + \tau_{\mathrm{H}})\left(R - \frac{B}{\Delta p + \Delta\tau}\right) > p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right).$$

This condition is necessarily satisfied if $\Delta\tau \geqslant 0$ (complementarity or separability). But it may fail if $\Delta\tau$ is sufficiently negative.

Exercise 10.7 (extent of control). The NPV is larger under limited investor control:

$$(p_{\mathrm{H}} + \tau_{\mathrm{A}})R - \gamma_{\mathrm{A}} > (p_{\mathrm{H}} + \tau_{\mathrm{B}})R - \gamma_{\mathrm{B}}.$$

We will assume that these NPVs are positive.

So the entrepreneur will grant limited control as long as this suffices to raise funds, i.e.,

$$(p_H + \tau_A)\left(R - \frac{B}{\Delta p}\right) \geqslant I - A.$$

If this condition is not satisfied, the entrepreneur must grant extended control in order to obtain financing. Financing is then feasible provided that

$$(p_H + \tau_B)\left(R - \frac{B}{\Delta p}\right) \geqslant I - A.$$

Lastly, note that

$$\tau_A R - \gamma_A \geqslant 0$$

is a sufficient condition for ruling out entrepreneurial control (but entrepreneurial control may be suboptimal even if this condition is not satisfied; for, it may conflict with the investors' breakeven condition).

Exercise 10.8 (uncertain managerial horizon and control rights). (i) The assumption

$$(p_H + \tau)\left(\frac{B}{\Delta p}\right) \geqslant \gamma$$

means that the new manager is willing to take on the job even if control is allocated to investors. Because his reward R_b must satisfy

$$(\Delta p)R_b \geqslant B,$$

regardless of who has control, the new manager receives rent

$$(p_H + \tau y)\left(\frac{B}{\Delta p}\right) - \gamma y$$

(smaller than the rent, $p_H B/\Delta p$, that he would receive if he were given control rights).

The entrepreneur's utility is (if the project is undertaken)

$$U_b = (1-\lambda)[(p_H + \tau x)R - \gamma x] + \lambda(p_H + \tau y)\left(R - \frac{B}{\Delta p}\right) - I.$$

The financing condition is

$$(1-\lambda)(p_H + \tau x)\left(R - \frac{B}{\Delta p}\right) + \lambda(p_H + \tau y)\left(R - \frac{B}{\Delta p}\right) \geqslant I - A.$$

(ii) Clearly, $y = 1$ both maximizes U_b and facilitates financing.

Also, a necessary condition for U_b to be positive is that λ not be too big.

Letting $\rho_0 \equiv p_H[R - B/\Delta p]$, if financing is feasible for $x = 0$: $(1-\lambda)\rho_0 + \lambda\rho_0^+ \geqslant I - A$, then $x = 0$ is optimal. The entrepreneur invests if and only if $U_b \geqslant 0$, or

$$(1-\lambda)\rho_1 + \lambda\rho_0^+ \geqslant I.$$

If $(1-\lambda)\rho_0 + \lambda\rho_0^+ < I - A$, then, in order to obtain financing, the entrepreneur must set x in the following way:

$$(1-\lambda)\rho_0 + \lambda\rho_0^+ + \tau x\left(R - \frac{B}{\Delta p}\right) = I - A.$$

Financing then occurs if and only if $U_b \geqslant 0$ for this value of x.

Exercise 10.9 (continuum of control rights). (i) Let R_b denote the entrepreneur's reward in the case of success. The entrepreneur maximizes her utility, which is equal to the NPV,

$$\max_{\{x(\cdot,\cdot)\}} \{[p_H + E_F[tx(t,g)]]R - I - E_F[gx(t,g)]\},$$

subject to the constraint that investors break even,

$$[p_H + E_F[tx(t,g)]][R - R_b] \geqslant I - A,$$

and to the incentive compatibility constraint,

$$(\Delta p)R_b \geqslant B.$$

Clearly, $R_b = B/\Delta p$ if the investors' breakeven constraint is binding. Let μ denote the shadow price of this constraint. Writing the Lagrangian and taking the derivative with respect to $x(t,g)$ for all t and g yields

$$x(t,g) = 1 \iff tR - g + \mu\left[t\left(R - \frac{B}{\Delta p}\right)\right] \geqslant 0.$$

This defines a straight line through the origin in the (t,g)-space under which $x = 1$ and over which $x = 0$.

(ii) When A decreases, more pledgeable income must be harnessed. So the straight line must rotate counterclockwise (add $t > 0$ realizations and subtract $t < 0$ ones). In the process, both τ and γ increase.

(iii) If $x(t,g) = 1$ and $t > 0$, the control right can be given to investors. If $x(t,g) = 1$ and $t < 0$ (which implies $g < 0$: the decision yields a private benefit to the entrepreneur), then the control can be allocated to the entrepreneur. Because

$$|g| > |t|R > |t|R_{\mathrm{b}},$$

the entrepreneur chooses $x(t,g) = 1$. Furthermore, $x(t,g) = 1$ is not renegotiated since it is first-best efficient.

One proceeds similarly for $x(t,g) = 0$.

(iv) Assume that g is the same for all rights and is positive. The optimal rule becomes

$$t \geqslant t^* = \frac{g}{R + \mu(R - B/\Delta p)}.$$

Let $H(t)$ denote the cumulative distribution function over t:

$$y \equiv g[1 - H(t^*)],$$
$$\tau \equiv \int_{t^*}^{\infty} t \, \mathrm{d}H(t).$$

Hence,

$$\frac{\mathrm{d}y}{\mathrm{d}\tau} = \frac{g}{t^*} \quad \text{and} \quad \frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} > 0.$$

One can, as earlier, envision that τ increases as A decreases, for example.

Exercise 12.1 (Diamond–Dybvig model in continuous time). To provide consumption $c(t)$ to consumers whose liquidity need arises between t and $t + \mathrm{d}t$ (in number $f(t)\,\mathrm{d}t$), one must cut $x(t)\,\mathrm{d}t$, where

$$x(t)R(t)\,\mathrm{d}t = c(t)f(t)\,\mathrm{d}t.$$

Together with the fact that the total number of trees per representative depositor is 1, this implies that the first-best contract solves

$$\max \left\{ \int_0^1 u(c(t))f(t)\,\mathrm{d}t \right\}$$
s.t.
$$\int_0^1 \frac{c(t)}{R(t)} f(t)\,\mathrm{d}t \leqslant 1.$$

The first-order condition is then, for each t,

$$\left[u'(c(t)) - \frac{\mu}{R(t)} \right] f(t) = 0,$$

where μ is the shadow price of the constraint.

(ii) Take the (log-) derivative of the first-order condition:

$$u'(c(t))R(t) = \mu \implies c\frac{u''}{u'}\frac{\dot{c}}{c} + \frac{\dot{R}}{R} = 0.$$

Because the coefficient of relative risk aversion exceeds 1,

$$\frac{\dot{c}}{c} < \frac{\dot{R}}{R}.$$

Note that, from the constraint, the average c/R is equal to 1. The existence of t^* follows (drawing a diagram may help build intuition).

(iii) Suppose that a depositor who has not yet suffered a liquidity shock withdraws at date τ. Reinvesting in the technology, she will obtain $c(\tau)R(t - \tau)$ if the actual date of the liquidity shock is $t > \tau$. Withdrawing is a "dominant strategy" (that is, yields more regardless of the future events) if

$$c(\tau)R(t - \tau) > c(t) \quad \text{for all } t > \tau.$$

The log-derivative of $(c(\tau)R(t - \tau)/c(t))$ with respect to t is, for τ close to 0,

$$\frac{\dot{R}(t-\tau)}{R(t-\tau)} - \frac{\dot{c}(t)}{c(t)} \simeq \frac{\dot{R}(t)}{R(t)} - \frac{\dot{c}(t)}{c(t)} > 0.$$

We thus conclude that the first-best outcome is not incentive compatible.

Exercise 12.2 (Allen and Gale (1998)[13] on fundamentals-based panics). (i) Let i_1 and i_2 denote the investments in the short- and long-term technologies. The social optimum solves

$$\max E[\lambda u(c_1(R)) + (1 - \lambda)u(c_2(R))]$$
s.t.
$$\lambda c_1(R) \leqslant i_1,$$
$$(1 - \lambda)c_2(R) \leqslant (i_1 - \lambda c_1(R)) + Ri_2,$$
$$i_1 + i_2 = 1.$$

This yields

(a) $c_1(R) = c_2(R) = i_1 + Ri_2$
$$\text{for } R \leqslant \frac{(1-\lambda)i_1}{\lambda i_2} = R^*,$$

(b) $c_1(R) = c_1(R^*)$,
and
$$c_2(R) = \frac{Ri_2}{1 - \lambda} \geqslant c_1(R) \quad \text{for } R \geqslant R^*.$$

For low long-term payoffs, $\lambda c_1(R) < i_1$ and the impatient types share risk with the patient types, as their short-term investment can be rolled over to

13. Allen, F. and D. Gale. 1998. Optimal financial crises. *Journal of Finance* 53:1245–1283.

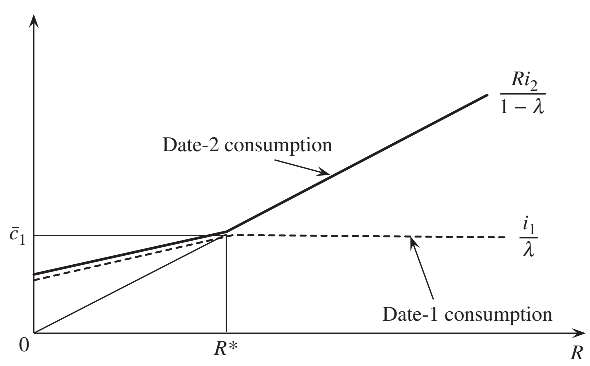

Figure 6

give some income to the latter. In contrast, high long-term payoffs (for which $\lambda c_1(R) = i_1$) are enjoyed solely by the patient types, who have no means of sharing the manna from heaven with the impatient types.

The optimal allocation is depicted in the Figure 6.

(ii) Let

$$\bar{c}_1 \equiv \frac{i_1}{\lambda}.$$

Suppose that the deposit contract promises

$$\min\{\bar{c}_1, i_1/(\lambda + (1-\lambda)x)\}$$

and that a fraction $x(R)$ of the patient depositors withdraw at date 1. For $R \leqslant R^*$, we claim that the following equations characterize the equilibrium:

$$[\lambda + (1-\lambda)x(R)]c_1 = i_1,$$
$$\frac{Ri_2}{(1-\lambda)(1-x(R))} = c_2,$$
$$c_1 = c_2.$$

First, note that, for $R > 0$, $x(R) = 1$ is not an equilibrium behavior, as patient consumers could consume an infinite amount by not withdrawing. Similarly, for $R < R^*$, $x(R) = 0$ is not part of an equilibrium because $Ri_2/(1-\lambda)$ is smaller than i_1/λ. Hence, a fraction in $(0,1)$ of patient consumers must withdraw at date 1. This implies that patient consumers are indifferent between withdrawing and consuming, or

$$c_1 = c_2.$$

Exercise 12.4 (random withdrawal rate). (i) This follows along standard lines. Asset maturities should match those of consumptions, $\lambda c_1 = i_1$ and $(1-\lambda)c_2 = i_2 R$:

$$\max_{c_1} \left\{\lambda u(c_1) + (1-\lambda)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right\}$$

implies

$$u'(c_1) = Ru'(c_2).$$

For CRRA utility, $c_1/c_2 = R^{-1/\gamma}$. So i_1 grows and i_2 decreases as risk aversion (γ) increases.

(ii) The optimal program solves

$$\max_{\{i_1,i_2,y_\cdot,z_\cdot\}} \left\{\beta\left[\lambda_L u\left(\frac{i_1 y_L + i_2 z_L \ell}{\lambda_L}\right) + (1-\lambda_L)u\left(\frac{i_1(1-y_L) + i_2 R(1-z_L)}{1-\lambda_L}\right)\right] + (1-\beta)\left[\lambda_H u\left(\frac{i_1 y_H + i_2 z_H \ell}{\lambda_H}\right) + (1-\lambda_H)u\left(\frac{i_1(1-y_H) + i_2 R(1-z_H)}{1-\lambda_H}\right)\right]\right\}.$$

Clearly, $z_\omega > 0 \Rightarrow y_\omega = 1$ and $y_\omega < 1 \Rightarrow z_\omega = 0$.

Also, $y_L = 1$ implies $y_H = 1$, and $z_H = 0$ implies $z_L = 0$.

For $\ell = 0$, the optimum has $z_\omega = 0$. It may be optimal to roll over some of i_1 in state L. For ℓ close to 1, i_2 serves to finance date-1 consumption in state H.

Exercise 13.1 (improved governance). (i) The pledgeable income is $p_H(R - B/\Delta p)$. The financing constraint is

$$(1+r)(I-A) \leqslant p_H\left(R - \frac{B}{\Delta p}\right).$$

(ii) The cutoff A^* is given by

$$(1+r)(I-A^*) = p_H\left(R - \frac{B}{\Delta p}\right).$$

Market equilibrium:

$$\left[S(r) + \int_0^{A^*(r)} Ag(A)\,\mathrm{d}A\right] = \int_{A^*(r)}^{I} (I-A)g(A)\,\mathrm{d}A$$

or, equivalently,

$$S(r) + \int_0^I Ag(A)\,\mathrm{d}A = [1 - G(A^*(r))]I.$$

(Note that entrepreneurs with weak balance sheets, $A < A^*$, would demand a zero rate of interest from their preferences. However, they receive the equilibrium market rate.)

Because A^* increases with the interest rate and with the quality of investor protection (here, $-B$), an increase in investor protection raises the equilibrium interest rate.

Exercise 13.2 (dynamics of income inequality). (i) See Section 13.3:

$$U_t(y_t) = y_t.$$

(ii) The incentive constraint is

$$(\Delta p)R_b^t \geqslant BI_t,$$

and so the pledgeable income is

$$p_H\left(R - \frac{B}{\Delta p}\right)I_t = \rho_0 I_t,$$

yielding an investment level given by

$$\rho_0 I_t = (1+r)(I_t - A_t) \quad \text{or} \quad I = \frac{A_t}{1-\rho_0/(1+r)}.$$

A project's NPV is

$$[p_H R - (1+r)]I_t = [\rho_1 - (1+r)]I_t.$$

By assumption, $\rho_1 \geqslant 1+r$, and so entrepreneurs prefer to invest in a project rather than lending their assets. Income is

$$y_t = [\rho_1 - \rho_0]I_t,$$

and so

$$A_{t+1} = a\frac{\rho_1 - \rho_0}{1-\rho_0/(1+r)}A_t + \hat{A},$$

which converges to A_∞ as t tends to ∞.

(iii) The threshold is given by

$$\frac{A_0^*}{1-\rho_0/(1+r)} = \underline{I}.$$

The limit wealth of poor dynasties is the limit point of the following first-order difference equation:

$$A_{t+1} = a(1+r)A_t + \hat{A}$$

or

$$A_\infty^L = \frac{\hat{A}}{1-a(1+r)}.$$

(iv) • If $\rho_1 = 1+r$, individuals are indifferent between being investors and becoming entrepreneurs. Note that wealths are equalized at

$$A_\infty = \frac{\hat{A}}{1-a\rho_1},$$

corresponding to investment

$$I_\infty = \frac{A_\infty}{1-\rho_0/(1+r)} = \frac{\rho_1\hat{A}}{(1-a\rho_1)(\rho_1-\rho_0)}.$$

Equilibrium in the loan market requires that

$$\kappa A_\infty = (1-\kappa)(I_\infty - A_\infty)$$

or

$$\kappa(\rho_1 - \rho_0) = (1-\kappa)\rho_0.$$

• If $\rho_1 > (1+r)$, then lenders must be unable to become entrepreneurs and so have wealth A_∞^L. Thus

$$\kappa A_\infty^L = (1-\kappa)(I_\infty - A_\infty),$$

where I_∞ was derived in question (ii).

Exercise 13.3 (impact of market conditions with and without credit rationing). (i) The representative entrepreneur's project has NPV (equal to the entrepreneur's utility)

$$U_b = p_H P R(I) - I - K,$$

and the scale of investment I can be financed as long as the pledgeable income exceeds the investors' initial outlay:

$$\mathcal{P}(I) \equiv p_H\left[PR(I) - \frac{BI}{\Delta p}\right] \geqslant I + K - A$$

(this is the financing condition).

In the absence of any financing constraint (i.e., when $B = 0$), the representative entrepreneur would choose a first-best (FB) policy:

$$p_H P R'(I^{FB}) = 1 \quad \text{or} \quad p_H P\alpha(I^{FB})^{\alpha-1} = 1,$$

provided that the fixed cost K is not too large, i.e., $K \leqslant p_H P R(I^{FB}) - I^{FB}$. (Otherwise, the optimal investment is equal to 0.)

When does the financing constraint bind?

Simple computations show that

$$\mathcal{P}(I^{FB}) - I^{FB} = (1-\alpha)\left[\frac{1}{\alpha} - \frac{p_H B/\Delta p}{1-\alpha}\right]I^{FB}.$$

Let us assume that the agency cost is not too large:

$$\frac{p_H B}{\Delta p} < \frac{1-\alpha}{\alpha}$$

(otherwise the financing constraint is necessarily binding).

Because I^{FB} is increasing in the product price P, the financing constraint is binding for low prices, as illustrated in Figure 7, where I^{SB} denotes the solution to the financing condition (taken with equality).

(ii) Thus, there is at least some region (to the left of P_0 in the figure) in which the expansionary impact of the product price (the contractionary impact of past investment) is stronger in the presence of credit rationing, i.e., when the presence of B makes the financing condition binding.

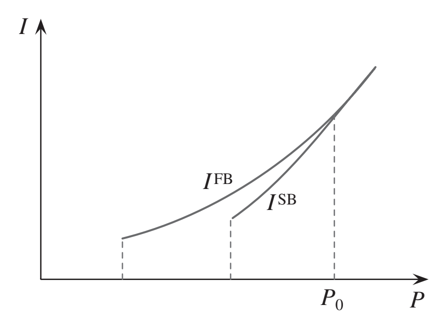

Figure 7

(iii) To conclude this brief analysis, we can now endogenize the product price by assuming the existence of a prior investment I_0 by, say, a mass 1 of the previous generation of entrepreneurs. Then, P is a decreasing function of total effective investment, i.e., total output:

$$P = P(p_{\text{H}}[R(I) + R(I_0)]), \quad \text{with } P' < 0.$$

When I_0 increases, I must decrease (if I increases, then P decreases, and so I decreases after all): this is the crowding-out effect; furthermore, total output must increase (if it decreased, then P would increase and so would I; and thus $p_{\text{H}}[R(I) + R(I_0)]$ would increase after all).

Exercise 14.2 (alternative distributions of bargaining power in the Shleifer–Vishny model). Entrepreneur i's utility (or, equivalently, firm i's NPV) is

$$\begin{aligned} U_{\text{b}i} &= [x\rho_1 + (1-x)(1-\nu)P - 1]I_i \\ &\quad + x(1-\mu)[(\rho_1 - \rho_0) + (\rho_0 - P)]I_j \\ &\equiv \hat{\alpha} I_i + \hat{\kappa} I_j, \end{aligned}$$

where

$$\hat{\alpha} = \alpha - (1-x)(1-\nu)(\rho_0 - P)$$

and

$$\hat{\kappa} = \kappa + x(1-\mu)(\rho_0 - P).$$

Recalling that $(1-x)(1-\nu) = x(1-\mu)$, note that $\hat{\alpha} + \hat{\kappa} = \alpha + \kappa$, as it should be from the fact that a change in bargaining power induces a mere redistribution of wealth for given investments.

Firm i's borrowing capacity is now given by

$$\begin{aligned} &[x\rho_0 + (1-x)(1-\nu)P]I_i \\ &\qquad + x(1-\mu)(\rho_0 - P)I_j = I_i - A_i \end{aligned}$$

or

$$I_i = \frac{A_i + x(1-\mu)(\rho_0 - P)I_j}{\begin{aligned} 1 + (1-x)(1-\nu)(\rho_0 - P) \\ -\rho_0[x + (1-x)(1-\nu)] \end{aligned}}.$$

In *symmetric* equilibrium ($A_1 = A_2 = A$; $I_1 = I_2 = I$),

$$I = \frac{A}{1 - \rho_0[x + (1-x)(1-\nu)]}$$

is independent of P.

Exercise 14.3 (liquidity management and acquisitions). (i) Suppose that the acquirer expects price demand P for the assets when the risky firm is in distress (which has probability $1-x$). The NPV for a given cutoff ρ^* is given by

$$U_{\text{b}}^{\text{s}} = (\rho_1 - 1)I + (1-x)J \int_0^{\rho^*} [\rho_1 - (P + \rho)]\,\mathrm{d}F(\rho).$$

The borrowing capacity in turn is given by

$$\rho_0 I + (1-x)J \int_0^{\rho^*} [\rho_0 - (P + \rho)]\,\mathrm{d}F(\rho) = I - A.$$

And so

$$\begin{aligned} U_{\text{b}}^{\text{s}} &= (\rho_1 - 1)\frac{A - (1-x)J\int_0^{\rho^*}[(P+\rho) - \rho_0]\,\mathrm{d}F(\rho)}{1 - \rho_0} \\ &\quad + (1-x)J \int_0^{\rho^*} [\rho_1 - (P + \rho)]\,\mathrm{d}F(\rho). \end{aligned}$$

Maximizing with respect to ρ^* and simplifying yields

$$\rho^* = 1 - P.$$

And so

$$\rho_0 + L^* = P + \rho^* = 1.$$

(ii) Anticipating that the safe firm has extra liquidity L^*, the seller chooses price P so as to solve

$$\max_P \{F(\rho_0 + L^* - P)P\},$$

since the acquirer can raise funds only when $P + \rho \leqslant \rho_0 + L^*$.

The derivative of this objective function is

$$-f(\rho^*)P + F(\rho^*) = -f(1-P)P + F(1-P).$$

Note that this derivative is positive at $P = 0$ and negative at $P = 1$. Furthermore, $-P + F(1-P)/f(1-P)$ is a decreasing function of P from the monotone hazard rate condition and so the equilibrium price is unique and belongs to $(0, 1)$.

Suppose next that L increases for some reason (and that this is observed by the seller). The first-order condition then becomes

$$-P + \frac{F(\rho_0 + L - P)}{f(\rho_0 + L - P)} = 0$$

and so

$$-\left[1 + \left(\frac{F}{f}\right)'\right]\frac{\mathrm{d}P}{\mathrm{d}L} + \left(\frac{F}{f}\right)' = 0.$$

Because $(F/f)' > 0$,

$$0 < \frac{\mathrm{d}P}{\mathrm{d}L} < 1.$$

This implies that the cutoff, and thus the probability of a sale, increases despite the price adjustment.

(iii) Suppose that the distribution F converges to a spike at $\bar{\rho}$. Consider thus a sequence $F_n(\rho)$ with

$$\lim_{n\to\infty} F_n(\rho) = 0 \quad \text{for } \rho < \bar{\rho}$$

and

$$\lim_{n\to\infty} F_n(\rho) = 1 \quad \text{for } \rho > \bar{\rho}.$$

Let us give an informal proof of the result stated in (iii) of the question. Choosing a price P that triggers a cutoff that is smaller than $\bar{\rho}$ and does *not* converge with n to $\bar{\rho}$ would yield (almost) zero profit, and so choosing an alternative price that leads to a cutoff a bit above $\bar{\rho}$ would yield a higher profit. Conversely, if the cutoff is above $\bar{\rho}$ and does *not* converge to $\bar{\rho}$, then $Pf_n \simeq 0$ and $F_n \simeq 1$, and so the first-order condition is not satisfied. (This proof is loose. A proper proof must consider a subsequence having the former or latter property.)

Exercise 14.4 (inefficiently low volume of asset reallocations). At the optimum, firm 1's assets are resold in the secondary market if and only if

$$\rho_0 < \rho_0^*.$$

Furthermore, it is optimal for the contract to specify that the proceeds from the sale to firm 2 go to the investors in firm 1 (so as to maximize the pledgeable income). And so the investment I is given by the investors' breakeven constraint:

$$\left[F(\rho_0^*)\hat{\rho}_0 + \int_{\rho_0^*}^{\hat{\rho}_0} \rho_0 \,\mathrm{d}F(\rho_0)\right] I = I - A,$$

which yields

$$I = I(\rho_0^*).$$

The entrepreneur's utility is

$$\begin{aligned} U_\mathrm{b} &= \mathrm{NPV} \\ &= \left[F(\rho_0^*)\hat{\rho}_0 + \int_{\rho_0^*}^{\hat{\rho}_0} (\rho_0 + \Delta\rho)\,\mathrm{d}F(\rho_0)\right] I(\rho_0^*). \end{aligned}$$

The optimal cutoff maximizes U_b and satisfies

$$\hat{\rho}_0 - \Delta\rho < \rho_0^* < \hat{\rho}_0.$$

Exercise 15.1 (downsizing and aggregate liquidity). (i) The incentive constraint is

$$(\Delta p)R_\mathrm{b}^0 \geqslant BI$$

in the case of no shock, and

$$(\Delta p)R_\mathrm{b}^\rho \geqslant BJ$$

in the presence of a liquidity shock.

So the pledgeable incomes are $p_\mathrm{H}(R(I) - BI/\Delta p)$ and $p_\mathrm{H}(R(J) - BJ/\Delta p)$, respectively.

The investors' breakeven constraint is

$$\begin{aligned} (1-\lambda)p_\mathrm{H}&\left[R(I) - \frac{BI}{\Delta p}\right] \\ &+ \lambda\left[p_\mathrm{H}\left[R(J) - \frac{BJ}{\Delta p}\right] - \rho J\right] \geqslant I - A. \quad (1) \end{aligned}$$

The entrepreneur's utility is equal to the NPV:

$$U_\mathrm{b} = (1-\lambda)p_\mathrm{H}R(I) + \lambda[p_\mathrm{H}R(J) - \rho J] - I. \quad (2)$$

Let μ denote the shadow price of constraint (1). Maximizing U_b subject to (1) (and ignoring the constraint $J \leqslant I$) yields first-order conditions with respect to I and J:

$$[(1-\lambda)p_\mathrm{H}R'(I) - 1][1+\mu] - \mu(1-\lambda)p_\mathrm{H}\frac{B}{\Delta p} = 0$$

or

$$p_\mathrm{H}R'(I) = \frac{1}{1-\lambda} + \frac{\mu}{1+\mu}p_\mathrm{H}\frac{B}{\Delta p}, \quad (3)$$

and

$$\lambda[p_\mathrm{H}R'(J) - \rho][1+\mu] - \lambda\mu p_\mathrm{H}\frac{B}{\Delta p} = 0$$

or

$$p_\mathrm{H}R'(J) \equiv \rho + \frac{\mu}{1+\mu}p_\mathrm{H}\frac{B}{\Delta p}. \quad (4)$$

Comparing (3) and (4), one observes that ignoring the constraint $J \leqslant I$ is justified if and only if

$$\rho > \frac{1}{1-\lambda},$$

that is, when the cost of continuation in the state of nature with a liquidity shock exceeds the cost of one more unit of investment in the state without. This simple comparison comes from the fact that the per-unit agency cost is the same in both states of nature. Let (I^*, J^*) denote the solution (obtained from (1), (3), and (4)).

(ii) • Under perfect correlation, no inside liquidity is available. So, in order to continue in the case of a liquidity shock, each firm requires

$$L = \rho J^*.$$

Hence, $L^* = \rho J^*$.

• If $L < L^*$, then

$$J = \frac{L}{\rho} < J^*. \tag{5}$$

• The solution is obtained by solving the modified program in which the extra cost associated with the liquidity premium, $(q-1)\rho J$, is subtracted in U_{b} (in (2)), and added to the right-hand side of (1), yielding a modified investor breakeven constraint—let us call it (1′). Equation (3) is unchanged, while (4) becomes

$$p_{\mathrm{H}} R'(J) = \rho\left(1 + \frac{q-1}{\lambda}\right) + \frac{\mu}{1+\mu} p_{\mathrm{H}} \frac{B}{\Delta p}. \tag{4′}$$

So $J < I$ *a fortiori*.

The liquidity premium is obtained by solving (1′), (3), (4′), and (5).

(iii) • Under independent shocks, exactly a fraction λ of firms incur no shock. Assuming $q = 1$ for the moment, (1) yields (provided $I > A$)

$$V = (1-\lambda) p_{\mathrm{H}}\left[R(I) - \frac{BI}{\Delta p}\right] + \lambda p_{\mathrm{H}}\left[R(J) - \frac{BJ}{\Delta p}\right] > \lambda \rho J. \tag{6}$$

V is the value of the stock index after the shocks have been met. And so the corporate sector, as a whole, can by issuing new claims raise enough cash to meet average shock $\lambda\rho J$. So there is, in principle, no need for outside liquidity.

• This, however, assumes that liquidity is not wasted. If each entrepreneur holds the stock index, then, when facing a liquidity shock, the entrepreneur can raise $p_{\mathrm{H}}[R(J) - BJ/\Delta p]$ by issuing new claims on the firm.

Meeting the liquidity shock then requires that

$$p_{\mathrm{H}}\left[R(J) - \frac{BJ}{\Delta p}\right] + [V - \lambda\rho J] \geqslant \rho J$$

or

$$(1-\lambda) p_{\mathrm{H}}\left[R(I) - \frac{BI}{\Delta p}\right] \geqslant (1+\lambda)\left[\rho J - p_{\mathrm{H}}\left[R(J) - \frac{BJ}{\Delta p}\right]\right],$$

which is not guaranteed.

It is then optimal to pool the liquidity, for example, through a credit line mechanism.

Exercise 15.2 (news about prospects and aggregate liquidity).

(i) $$\mathrm{NPV} = \int_{y^*}^{1} y \,\mathrm{d}G(y) - [1 - G(y^*)]J - I.$$

Investors' net income

$$= \int_{y^*}^{1} y \,\mathrm{d}G(y) - [1 - G(y^*)][J + \mathcal{R}] - [I - A].$$

(ii) • The NPV is maximized for $y^* = y_0^* = J$. So, if

$$\int_{J}^{1} y \,\mathrm{d}G(y) - [1 - G(J)][J + \mathcal{R}] \geqslant I - A \iff A \geqslant A_0^*,$$

then $y^* = J$.

Otherwise, by concavity of the NPV, the contract raises y^* so as to attract investment:

$$\int_{y^*}^{1} y \,\mathrm{d}G(y) - [1 - G(y^*)][J + \mathcal{R}] = I - A.$$

The pledgeable income can no longer be increased when $y^* = y_1^* = J + \mathcal{R}$.

So, for $A < A_1^*$, no financing is feasible.

• If $A > A_1^*$, then $y^* < J + \mathcal{R}$. Hence, for $y^* \leqslant y < J + \mathcal{R}$, investors have negative profit from continuation, and the firm cannot obtain financing just by going back to the capital market.

(iii) If productivities are drawn independently, the financing constraint,

$$\int_{y^*}^{1} y \,\mathrm{d}G(y) - [1 - G(y^*)][J + \mathcal{R}] = I - A,$$

implies

$$\int_{y^*}^{1} y \,\mathrm{d}G(y) - [1 - G(y^*)][J + \mathcal{R}] > 0,$$

and so, *collectively*, firms have enough income to pledge when going back to the capital market.

(iv) • Suppose, in a first step, that there exists a large enough quantity of stores of value, and so

$q = 1$ (there is no liquidity premium). Then the breakeven condition can be written as

$$E_\theta\bigg[\int_{y^*(\theta)}^1 (y - J - \mathcal{R})\,\mathrm{d}G(y \mid \theta)\bigg] \geqslant I - A.$$

• Maximize

$$E_\theta\bigg[\int_{y^*(\theta)}^1 (y - J)\,\mathrm{d}G(y \mid \theta)\bigg] - I$$

subject to the financing constraint (let μ denote the multiplier of the latter). Then

$$y^*(\theta) - J + \mu[y^*(\theta) - J - \mathcal{R}] = 0$$
$$\Longrightarrow\ y^*(\theta) = J + \frac{\mu}{1+\mu}\mathcal{R}.$$

• The lowest amount of pledgeable income,

$$\min_{\{\theta\}} \int_{y^*}^1 (y - J - \mathcal{R})\,\mathrm{d}G\,(y \mid \theta),$$

may be negative. It must then be complemented by an equal number of stores of value delivering one for certain, say.

• If there are not enough stores of value, then they trade at a premium $(q > 1)$.

Exercise 15.3 (imperfectly correlated shocks). A shortage of liquidity may occur only if the fraction θ of correlated firms faces the high shock (the reader can follow the steps of Section 15.2.1 to show that in the other aggregate state there is no liquidity shortage).

The liquidity need is then, in aggregate,

$$[\theta + (1-\theta)\lambda](\rho_{\mathrm{H}} - \rho_0)I.$$

The net value of shares in the healthy firms is

$$(1-\theta)(1-\lambda)(\rho_0 - \rho_{\mathrm{L}})I.$$

Using the investors' breakeven condition and the assumption that liquidity bears no premium:

$$[(1-\lambda)(\rho_0 - \rho_{\mathrm{L}}) - \lambda(\rho_{\mathrm{H}} - \rho_0)]I = I - A.$$

And so the corporate sector is self-sufficient if

$$(1-\theta)(1-\lambda)(\rho_0 - \rho_{\mathrm{L}})I \geqslant [\theta + (1-\theta)\lambda](\rho_{\mathrm{H}} - \rho_0)I$$

or

$$(1-\theta)(I - A) \geqslant \theta(\rho_{\mathrm{H}} - \rho_0)I.$$

Exercise 15.4 (complementarity between liquid and illiquid assets). The NPV per unit of investment is equal to

$$(1 - \lambda + \lambda x)\rho_1$$
$$- [1 + (1-\lambda)\rho_{\mathrm{L}} + [\lambda\rho_{\mathrm{H}} + (q-1)(\rho_{\mathrm{H}} - \rho_0)]x].$$

We know that this NPV is negative for $x = 0$. Thus, either its derivative with respect to x is nonpositive,

$$\lambda\rho_1 \leqslant \lambda\rho_{\mathrm{H}} + (q-1)(\rho_{\mathrm{H}} - \rho_0),$$

and then there is no investment ($I = 0$). The absence of corporate investment implies that there is no corporate demand for liquidity, and so $q = 1$, which contradicts the fact that $\rho_1 > \rho_{\mathrm{H}}$. Hence, the derivative with respect to x must be strictly positive:

$$\lambda\rho_1 > \lambda\rho_{\mathrm{H}} + (q-1)(\rho_{\mathrm{H}} - \rho_0),$$

implying that $x = 1$.

For a low supply of liquid assets, this in turn implies that

(a) investment is limited by the amount of liquid assets,

$$L^{\mathrm{S}} = (\rho_{\mathrm{H}} - \rho_0)I;$$

(b) the entrepreneurs compete away the benefits associated with owning liquid assets, and so they are indifferent between investing in illiquid and liquid assets and not investing at all,

$$\rho_1 = 1 + \bar{\rho} + (\bar{\bar{q}} - 1)(\rho_{\mathrm{H}} - \rho_0).$$

Furthermore, for a low supply of liquid assets, entrepreneurs do not borrow as much as their borrowing capacity would allow them to. This borrowing capacity, denoted $\bar{I}$, is given by

$$\rho_0\bar{I} = [1 + \bar{\rho} + (\bar{\bar{q}} - 1)(\rho_{\mathrm{H}} - \rho_0)]\bar{I} - A$$
$$= \rho_1\bar{I} - A.$$

When L^{S} reaches $\underline{L}^{\mathrm{S}}$, given by

$$\underline{L}^{\mathrm{S}} \equiv \frac{\rho_{\mathrm{H}} - \rho_0}{\rho_1 - \rho_0}A,$$

then $I = \bar{I}$. For $L^{\mathrm{S}} > \underline{L}^{\mathrm{S}}$, q decreases with L^{S} and investment,

$$I = \frac{A}{1 + \bar{\rho} + (q-1)(\rho_{\mathrm{H}} - \rho_0) - \rho_0} = \frac{L^{\mathrm{S}}}{\rho_{\mathrm{H}} - \rho_0},$$

increases until $L^{\mathrm{S}} = \bar{L}^{\mathrm{S}}$ (i.e., $q = 1$), after which it is no longer affected by the supply of liquid assets.

Exercise 16.1 (borrowing abroad). (i) Investing abroad is inefficient since $\mu < 1$. So it is optimal to prevent investment abroad. Letting R_{l} denote the return to investors in the case of success, the incentive compatibility constraint is

$$p(RI - R_{\mathrm{l}}) \geqslant \mu I.$$

The breakeven constraint is

$$pR_{\mathrm{l}} = I - A.$$

The NPV,

$$U_{\mathrm{b}} = (pR - 1)I,$$

is maximized when I is maximized subject to the incentive compatibility and breakeven constraints, and so

$$I = \frac{A}{1-(pR-\mu)}, \quad \text{and so } U_{\mathrm{b}} = \frac{pR-1}{1-(pR-\mu)}A.$$

This is a reinterpretation of the basic model with

$$p_{\mathrm{H}} = p, \quad p_{\mathrm{L}} = 0, \quad B = \mu.$$

Investing abroad brings the probability of success of the domestic investment down to 0. And because investors are unable to grab any of the diverted funds, their proceeds are but a private benefit for the entrepreneur.

(ii) One has

$$p[(1-\tau)RI - R_{\mathrm{l}}] \geqslant \mu I$$

and

$$pR_{\mathrm{l}} + (1-p)\sigma R_{\mathrm{l}} = I - A.$$

The government's breakeven constraint is

$$p\tau RI = (1-p)\sigma R_{\mathrm{l}}.$$

The borrowing capacity is unchanged, because the pledgeable income is unaffected.

In contrast, when public debt D (per entrepreneur) is financed through corporate taxes,

$$p\tau RI = D,$$

then

$$I = \frac{A-D}{1-(pR-\mu)}$$

and

$$U_{\mathrm{b}} = \frac{pR-1}{1-(pR-\mu)}(A-D).$$

(iii) In the case of government commitment, $\mu = \mu_{\mathrm{L}}$ maximizes U_{b}. In the absence of commitment, suppose that investors expect $\mu = \mu_{\mathrm{L}}$. Then the entrepreneurs receive

$$p(RI - R_{\mathrm{l}}) = \mu_{\mathrm{L}} I \quad \text{if } \mu = \mu_{\mathrm{L}}$$

and

$$\max(p(RI - R_{\mathrm{l}}), \mu_{\mathrm{H}} I) = \mu_{\mathrm{H}} I \quad \text{if } \mu = \mu_{\mathrm{H}}.$$

Hence, $\mu = \mu_{\mathrm{H}}$. And U_{b} is decreased.

(iv) The exchange rate is given at date 2 by

$$e\mathcal{R} = pR_{\mathrm{l}}.$$

(Assuming that there is no excess supply of tradables $\mathcal{R}$; otherwise $e \equiv 1$.) One has

$$p(RI - R_{\mathrm{l}}) = \mu I$$

and

$$\frac{pR_{\mathrm{l}}}{e} = I - A.$$

Then

$$I = \mathcal{R} + A = \frac{A}{1-(pR-\mu)/e}.$$

$e \geqslant 1$ is equivalent to $(1 + A/\mathcal{R})(pR - \mu) \geqslant 1$.

Exercise 16.2 (time-consistent government policy). (i) The incentive constraint is

$$[(p_{\mathrm{H}} + \tau) - (p_{\mathrm{L}} + \tau)]R_{\mathrm{b}} \geqslant BI.$$

And so the investors' breakeven condition is

$$(p_{\mathrm{H}} + \tau)\left(R - \frac{B}{\Delta p}\right)I = I - A.$$

This yields $I(\tau)$.

The government maximizes

$$[(p_{\mathrm{H}} + \tau)R - \gamma(\tau)]I.$$

Hence,

$$\gamma'(\tau^*) = R.$$

(ii) $\max_\tau\{[(p_{\mathrm{H}} + \tau)R - 1 - \gamma(\tau)]I\}$

$$\Longrightarrow\ [\gamma'(\tau^c) - R]I = [(p_{\mathrm{H}} + \tau)R - 1 - \gamma(\tau^c)]\frac{\mathrm{d}I}{\mathrm{d}\tau}.$$

(iii) $\tau < \tau^*$ then.

Exercise 16.3 (political economy of exchange rate policies). (i) $d^* = p_H R_l^*$ and $d = p_H R_l^S + (1 - p_H) R_l^F$.

(ii) The entrepreneur's incentive constraint (expressed in tradables) is

$$(\Delta p)\left[R_b^* + \frac{R_b^S - R_b^F}{e}\right] \geqslant BI.$$

The foreign investors' breakeven constraint can be written as

$$d^* + \frac{d}{e} = p_H R_l^* + \frac{p_H R_l^S + (1 - p_H) R_l^F}{e} \geqslant I - A.$$

And so, adding up these two inequalities,

$$p_H\left(R - \frac{B}{\Delta p}\right)I + \frac{p_H SI + (1 - p_H) R_l^F - p_H R_b^F}{e} \geqslant I - A.$$

Thus, if the NPV per unit of investment is positive (which we will assume), it is optimal to set

$$R_b^F = 0 \quad \text{and} \quad R_l^F = SI.$$

The investment is therefore

$$I(e) = \frac{A}{1 - [(S/e) + \rho_0]}. \tag{1}$$

It decreases as the exchange rate depreciates because part of the firm's production is in nontradables.

(iii) *Commitment.* Suppose, first, that the government chooses g^* *before* entrepreneurs borrow abroad.

The representative entrepreneur has expected utility

$$[SI - d] + p_H R_b^* + \max_{c_1^*}[u(c_1^*) - ec_1^*] + v(g^*).$$

In the end, the entrepreneur's average consumption of nontradables is

$$SI$$

and the (average and individual) consumption of tradables is

$$\mathcal{R}^* - g^* + [p_H R - 1]I + A$$

since the NPV, $(p_H R - 1)I + SI$, must accrue to them from the investors' breakeven condition.

Hence, the government chooses g^* so as to solve

$$\max_{g^*}\{SI + u(\mathcal{R}^* - g^* + [p_H R - 1]I + A) + v(g^*)\}$$

subject to (1) and the market-clearing equation,

$$p_H RI(e) + \mathcal{R}^* - g^* = c_1^*(e) + [I(e) - A]. \tag{2}$$

The first-order condition is (using $u' = e$)

$$v'(g^*) = e\left[1 - \left[\frac{S}{e} + (p_H R - 1)\right]\frac{\mathrm{d}I}{\mathrm{d}e}\frac{\mathrm{d}e}{\mathrm{d}g^*}\right] > e.$$

Noncommitment. Under noncommitment, investment is fixed at some level $\bar{I}$ at the date at which g^* is chosen. So the government solves

$$\max_{g^*}\left\{S\bar{I} + u\left(\mathcal{R}^* - g^* + p_H R\bar{I} - d^* - \frac{d}{e}\right) + v(g^*)\right\}$$

and so

$$v'(g^*) = e\left[1 - \frac{d}{e^2}\frac{\mathrm{d}e}{\mathrm{d}g^*}\right] < e.$$

(iv) Note that under noncommitment g^* increases as the debt expressed in nontradables, d, increases. Overspending imposes a negative externality on foreigners when their claims are in nontradables and therefore can be depreciated.

Each borrower would be better off if the other borrowers issued fewer claims in nontradables. But each borrower also has an individual incentive to use nontradables as collateral so as to maximize borrowing capacity.